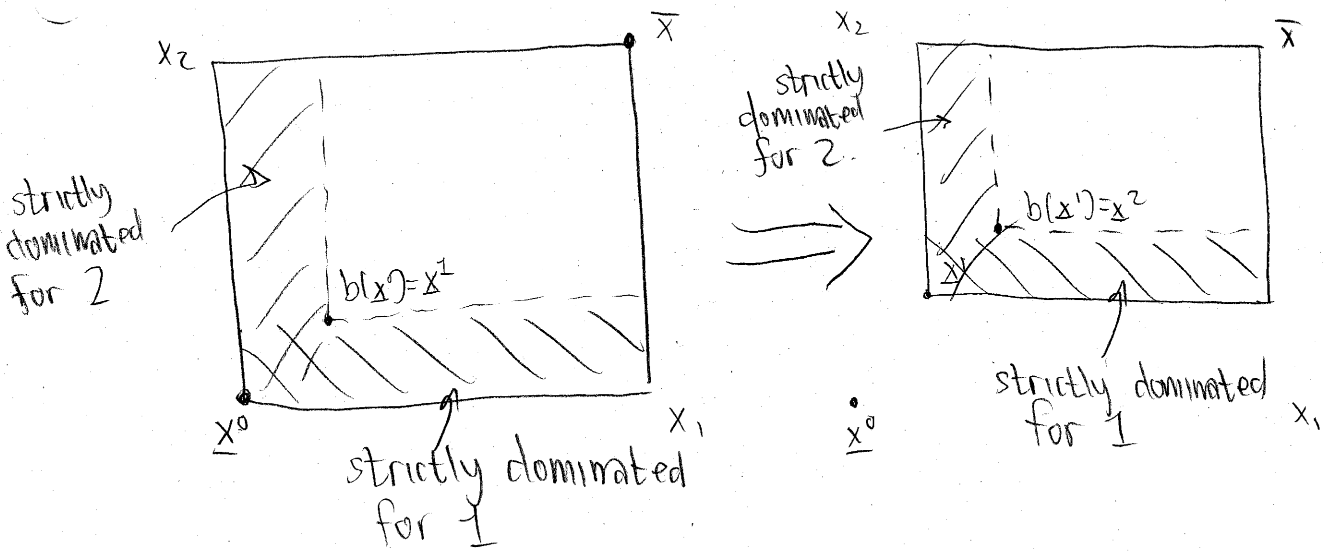


- $\underline{x}^k = b(\underline{x}^{k-1}) \Rightarrow \underline{x}^k \geq \underline{x}^{k-1} \geq \underline{x}^{k-2} \geq \dots$ since b is monotone.
- this is an increasing sequence, so since this is a complete lattice, it has a limit. That is, $\lim_{k\to\infty} \underline{x}^k = \sup\{\underline{x}^k : k > 0\}$

<u>Thm</u>: The smallest rationalizable strategies for the players are given by $\underline{z} = \lim_{k\to\infty} b^k(\underline{x})$. Similarly, the largest rationalizable strategies for the players are given by $\bar{z} = \lim_{k\to+\infty} B^k(\bar{x})$. Further, $\underline{z}$ and $\bar{z}$ are Nash equilibria

Pf that $\underline{z}$ is a NE:

Suppose that $\underline{z}$ is not a NE. Then $\exists n$ and $\exists x_n$ s.t. $u_n(x_n, \underline{z}_{-n}) > u_n(\underline{z}_n, \underline{z}_{-n})$. Consider $(x_n, \underline{x}_{-n}^k)$ and $(\underline{x}_n^k, \underline{x}_{-n}^{k-1})$

$\xrightarrow{k\to\infty} (x_n, \underline{z}_{-n})$ $\quad \xrightarrow{k\to\infty} (\underline{z}_n, \underline{z}_{-n})$

By continuity of the utility function,

$u_n(x_n, \underline{x}_{-n}^k) \to u_n(x_n, \underline{z}_{-n})$ and

$u_n(\underline{x}_n^k, \underline{x}_{-n}^{k-1}) \to u_n(\underline{z}_n, \underline{z}_{-n})$. Thus, $\exists k$ s.t.

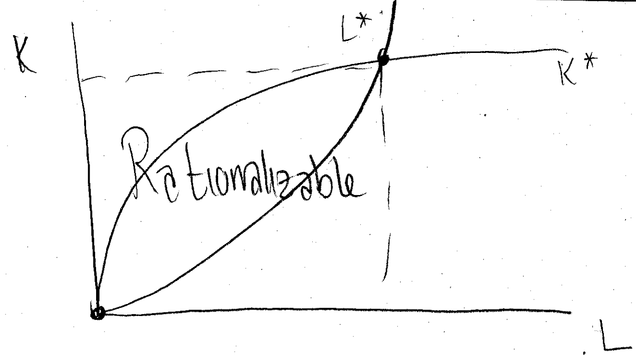
$u_n(x_n^k, \underline{x}_{-n}^k) > u_n(\underline{x}_n^k, \underline{x}_{-n}^{k-1})$

$\Rightarrow \underline{x}_n^k \notin BR_n(\underline{x}_{-n}^{k-1})$, which is a contradiction $\times$

Thus, $\underline{z}$ is a NE.

Partnership Game

- Two players E(mployer), W(orker)
- E provides K, W provides L
- $f(K,L) = K^\alpha L^\beta \qquad 0 < \alpha, \beta, \alpha+\beta < 1$
- Payoffs: $\frac{f(K,L)}{2} - K, \frac{f(K,L)}{2} - L$



Comparative Statics in Supermodular Games

Thm: Consider a family of supermodular games w/ payoffs parametrized by t. Suppose that $\forall n$, $\forall x_{-n}$, $u_n(x_n, x_{-n}, t)$ is supermodular in (x_n, t). Then,

$\bar{z}(t), \underline{z}(t)$ are isotone.

Pf: $b_n(x, t)$ is isotone in (x_{-n}, t)

Take t, t' s.t. $t \geq t'$. Then $\underline{x}^0(t) \equiv \underline{x}^0(t') \equiv \underline{x}$

$\Rightarrow \underline{x}^1(t) = b(\underline{x}^0(t), t) \geq b(\underline{x}^0(t'), t') = \underline{x}^1(t')$

$\Rightarrow \underline{x}^k(t) = b(\underline{x}^{k-1}(t), t) \underset{\uparrow}{\geq} b(\underline{x}^{k-1}(t'), t') = \underline{x}^k(t')$

inductive hypothesis

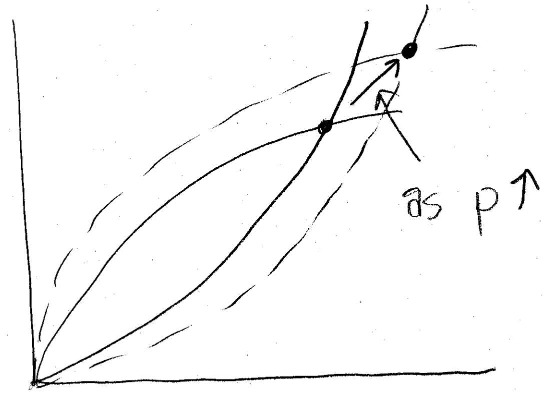
$$\Rightarrow \underbrace{\sup\{\underline{x}^k(t), k>0\}}_{= \lim_{k \to \infty} \underline{x}^k(t) = \underline{z}(t)} \geq \underbrace{\sup\{\underline{x}^k(t'), k>0\}}_{= \lim_{k \to \infty} \underline{x}^k(t') = \underline{z}(t')} \quad \square$$

Similarly for $\bar{z}$.

E.g. Partnership'

$f(K,L) = pK^{\alpha}L^{\beta}$

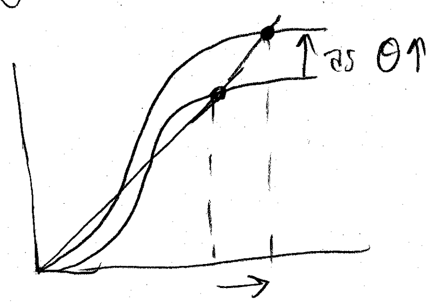
$\frac{\partial u}{\partial p} = K^{\alpha}L^{\beta}$ increasing in both K and L.



Peter Diamond Search model

- continuum of players
- each i puts effort a_i, costing $a_i^2/2$
- Pr i finds match: $a_i g(\underline{a}_{-i})$
- payoff: $u_i(a) = \theta a_i g(\underline{a}_{-i}) - a_i^2/2$

$\Rightarrow$



Monotone supermodular games of incomplete info.

- $G = (N, T, A, u, p)$
- $T = T_0 \times \dots \times T_n \subseteq \mathbb{R}^{M \times n}$
- A_i - compact sublattice of $\mathbb{R}^K$
- $u_i(a, \cdot): A \times T \to \mathbb{R}$
  - measurable

- $u_i(\cdot, t): A \to \mathbb{R}$ is continuous, bdd, supermodular in a_i, has increasing differences in a_i
  - ie for fixed t, this is a supermodular game.
- $p(\cdot | t_i)$ increasing in t_i (in FOSD sense)
  - ie p is affiliated.
  - if see high signal, think other guy had high signal.
  - e.g. $t_i = \theta + \varepsilon \eta_i \Rightarrow \theta | t_i \sim -\varepsilon \eta_i + t_i$

<u>Thm:</u> There exists BNE $\bar{s}, \underline{s}$ such that.
- For each BNE $\bar{s} \geq s \geq \underline{s}$
- Both $\bar{s}$ and $\underline{s}$ are isotone.

	I	NI
I	θ, θ	$\theta-1, 0$
NI	$0, \theta-1$	$0, 0$