Fudenberg, Levine, Maskin (Ema '94): Folk thm for imperfect public monitoring.

$\boxed{\text{I}}$ an example where the folk theorem "fails"

$\boxed{\text{II}}$ conditions for which it doesn't

$\boxed{\text{III}}$ pure strategy SPEs are outcome equivalent to PPE.

Example:

- 2 players, $i \in \{1,2\}$
- $e \in \{0,1\}$
- $y \in \{0, 12\}$
- $u_i(e_i; y) = \frac{1}{2}y - 3e$
- $y|(e_i, e_{-i})$ distribution of output conditional on effort

end result: when you can identify deviations from the dist. of y, then folk theorem will hold.

- $Pr[y=0 | (e_i, e_{-i}) = (1,1)] = \frac{1}{3}$
- $Pr[y=12 | (e_i, e_{-i}) = (1,1)] = \frac{2}{3}$

nevermind, those probs. are in the table below

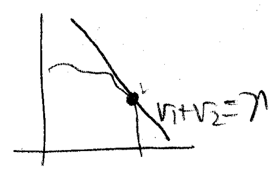
$Pr[y=12 \mid \cdot]$	$e_1 = 1$	$e_1 = 0$
$e_2 = 1$	2/3	1/3
$e_2 = 0$	1/3	0

$E[u_2]$ \ $E[u_1]$	$e_1=1$	$e_1=0$
$e_2=1$	1,1	-1,2
$e_2=0$	2,-1	0,0

UPPE(δ)

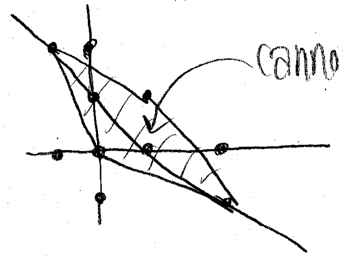
Recall: If v is sustainable by a SPE in pure strategies, then v is sustainable for some PPE,

Hence, if cannot get the folk theorem in PPE, you cannot get it in pure SPE.



Define $\eta = \max\{v_1+v_2 : (v_1, v_2) \in UPPE(\delta)\}$

∘ Result: $\eta \leq 1$



assume $\eta > 1$. Pick PPE that gives η

Let $\alpha = (\alpha_i, \alpha_{-i})$ be (potentially mixed) actions at h^0.

$v_i(0), v_i(12)$ continuation values ($i \in \{1,2\}$)

$$v_i^* = (1-\delta)u_i(\alpha) + \delta\{Pr[y=12|\alpha]\, v_i(12) + Pr[y=0|\alpha]\, v_i(0)\}$$

$$v_1^* + v_2^* = \eta$$

Step 1: α_1 must put prob >0 on $e_1=1$. Otherwise,

$$u_1(\alpha)+u_2(\alpha)\le 1 \Rightarrow v_1^*+v_2^*\le(1-\delta)1 + \delta\Big(\Pr[y=12|\alpha]\overbrace{(v_1(12)+v_2(12))}^{\le \gamma} + \Pr[y=0|\alpha]\underbrace{(v_1(0)+v_2(0))}_{\le\gamma}\Big)$$

by stationarity of PPE and defn of γ.

$$\Rightarrow \gamma \equiv v_1^*+v_2^* \le (1-\delta)\cdot\underbrace{1}_{<\gamma}+\delta\gamma < (1-\delta)\gamma+\delta\gamma=\gamma \rightarrow\leftarrow$$

Thus, α_i must put prob >0 on $e_i=1$.

Step 2: What does $\alpha_i(e_i=1)>0$ mean? It means it is weakly optimal for i to play $e_i=1$.

Define $\mu_2\equiv\alpha_2(e_2=1)$, $\mu_1\equiv\alpha_1(e_1=1)$

$\mu_1>0 \Rightarrow E[\text{payoff of } e_1=1] \ge E[\text{payoff of } e_1=0]$

$$\Leftrightarrow (1-\delta)(\mu_2+(1-\mu_2)(-1))+\delta\left[\left(\tfrac{2}{3}\mu_2+\tfrac{1}{3}(1-\mu_2)\right)v_1(12)+\left(\tfrac{1}{3}\mu_2+\tfrac{2}{3}(1-\mu_2)\right)v_1(0)\right]$$

$$=(1-\delta)(2\mu_2-1)+\delta\left[\left(\tfrac{1}{3}\mu_2+\tfrac{1}{3}\right)v_1(12)+\left(-\tfrac{1}{3}\mu_2+\tfrac{2}{3}\right)v_1(0)\right]$$

$$\ge \underbrace{(1-\delta)(2\mu_2)+\delta\left[\mu_2\tfrac{1}{3}v_1(12)+\left(\tfrac{2}{3}\mu_2+(1-\mu_2)\right)v_1(0)\right]}_{\text{payoff if shirk}}$$

$$=(1-\delta)2\mu_2+\delta\left[\mu_2\tfrac{1}{3}v_1(12)+\left(-\tfrac{1}{3}\mu_2+1\right)v_1(0)\right]$$

$$\Rightarrow -(1-\delta)+\delta\left[\tfrac{1}{3}v_1(12)-\tfrac{1}{3}v_1(0)\right]\ge 0$$

$$\Leftrightarrow v_1(12)\ge v_1(0)+\frac{3(1-\delta)}{\delta}$$

ie there needs to be a minimal gap between those continuation payoffs

Step 3: $v_1^* \le (1-\delta)u_1(\alpha) + \delta\left[\frac{2}{3}v_1(12) + \frac{1}{3}\left(v_1(12) - \frac{3(1-\delta)}{\delta}\right)\right]$

since $\Pr[y=12|\alpha] \le \frac{2}{3}$

$\le (1-\delta)u_1(\alpha) + (1-\delta) + \delta v_1(12)$

$v_2^* \le (1-\delta)u_2(\alpha) + (1-\delta) + \delta v_2(12)$

$$\Rightarrow \underbrace{v_1^* + v_2^*}_{=\gamma} = (1-\delta)\underbrace{(u_1(\alpha) + u_2(\alpha) - 2)}_{<0} + \delta\underbrace{(v_1(12) + v_2(12))}_{\le \gamma}$$

$\le \delta\gamma < \gamma$ $\rightarrow\leftarrow$

Thus, $\gamma \le 1$.

Identifiability

Condition for FT: $\forall \alpha = (\alpha_i, \alpha_{-i})$

1] α is individually identifiable if $y|\alpha_i, \alpha_{-i} \ne y|\alpha_i', \alpha_{-i}$ whenever $\alpha_i \ne \alpha_i'$

$$\Leftrightarrow \text{rank} \underbrace{\begin{bmatrix} \Pr[y=y_1|\alpha_1^i, \alpha_{-i}] & \cdots & \Pr[y=y_m|\alpha_1^i, \alpha_{-i}] \\ \vdots & & \\ \Pr[y=y_1|\alpha_{m_i}^i, \alpha_{-i}] & \cdots & \Pr[y=y_m|\alpha_{m_i}^i, \alpha_{-i}] \end{bmatrix}}_{\equiv \Pi_i} = m_i \ \forall i.$$

2] α is pairwise identifiable if $\forall j \ne i$, $\forall \alpha_i' \ne \alpha_i$, $\forall \alpha_j' \ne \alpha_j$, $y|\alpha_{-j}, \alpha_j' \ne y|\alpha_{-i}, \alpha_i'$