Reputation in Bargaining: screening

- Seller owns an object
- Buyer
- Value of object for
  - seller is 0
  - buyer is $v \geq 0$, private
- at each t,
  - seller sets a price p_t
  - buyer decides whether to buy at p_t
- If trade at t, payoffs are:
  - $\delta^t p_t$ for seller
  - $\delta^t (v - p_t)$ for buyer

In equilibrium, high value buyers buy at weakly earlier time

On equilibrium path, if buyer keeps not buying,
- seller puts higher and higher probability on the lower values
- buyer forms reputation for low value
- prices decrease

Coase: Let $\tau = t/k$ be real time, $\delta = e^{-r/k}$, as $k \to \infty$, in equilibrium

- the real time of trade goes to 0
- prices go to lowest possible value of v.

<u>an example</u>

- $v \sim U[0,1]$
- look for equilibrium s.t.
  - $\forall P_t$, buy iff $v > \lambda P_t$, where $\lambda > 1$
  - If $v \sim U[0,K]$ at a history, $P_t = \gamma K$, $\gamma \in (0,1)$
    - $K = \lambda P_t$
    - $P_{t+1} = \gamma \lambda P_t = (\gamma\lambda)^t P_0$
- $U_s(K)$ = seller's expected payoff times K.
- sequential rationality:
  - $U_s(K) = \max_p \{(K - \lambda P)P + \delta U_s(\lambda P)\}$
  - FOC and envelope theorem:

$$1 - 2\lambda\gamma + \delta\lambda^2\gamma^2 = 0$$

- Indifference at the cutoff:

<u>Today</u>	<u>Tomorrow</u>
$v - P_t$	$\delta(v - P_{t+1})$

$\Rightarrow v = \lambda p \Rightarrow$ these people are indifferent

- $\lambda p - p = \delta(\lambda p - \gamma\lambda p)$

$\circ\ \lambda - 1 = \delta\lambda - \eta\delta\lambda$

Thus, $\lambda = \frac{1}{(1-\delta)^{1/2}}$ $\eta = \frac{(1-\delta)^{1/2} - (1-\delta)}{\delta}$

as $\delta \to 1$, $\eta \to 0 \Rightarrow \beta_t = \eta k \to 0$ as $\delta \to 1$ $\forall t$.

Rationalizability in Incomplete Information Games

Common knowledge of rationality $\Leftrightarrow$ rationalizability
- stronger foundations for this than for equilibrium

Many evolutionary/learning processes converge to the rationalizable set
- if a process starts in equilibrium, it tends to stay. Convergence to equilibrium is hardly guaranteed.

Robust predictions of any equilibrium refinement $\Leftrightarrow$ predictions of rationalizability.

Defn: A Bayesian Game is (N, Θ, T, p, A, u)

Ex ante rationalizability
- Every incomplete information game (with $p_i(\cdot|t_i) = p(\cdot|t_i)$) is a complete information game with nature

- strategies: $s_i : T_i \to A_i$
- Utilities : $U_i(s) = \sum_{(\theta,t)} u_i(\theta, s(t))\, p(\theta, t)$

Ex ante rationalizability $\Leftrightarrow$ rationalizability in the resulting complete information game

Eg.

$\Theta = \{\theta, \theta'\}$

$T_1 = \{t_1, t_1'\}$, $T_2 = \{t_2\}$

$p(\theta, t_1, t_2) = p(\theta', t_1', t_2) = \frac{1}{2}$

θ	L	R
U	$1, \varepsilon$	$-2, 0$
D	$0, 0$	$0, 1$

θ'	L	R
U	$-2, \varepsilon$	$1, 0$
D	$0, 0$	$0, 1$

	L	R	
UU	~~$-\frac{1}{2}, \varepsilon$~~	~~$-\frac{1}{2}, 0$~~	Dom by DD (1)
UD	~~$\frac{1}{2}, \frac{\varepsilon}{2}$~~	~~$-\frac{1}{2}, \frac{1}{2}$~~	Dom by DU (3)
DU	~~$-\frac{1}{2}, \frac{\varepsilon}{2}$~~	$\frac{1}{2}, \frac{1}{2}$	
DD	~~$0, 0$~~	~~$0, 1$~~	Dom by DU (3)
	Dom by R (2)		

$U_1(UU, L) = \frac{1}{2}(1) + \frac{1}{2}(-2) = -\frac{1}{2}$

$U_1(UU, R) = \frac{1}{2}(-2) + \frac{1}{2}(1) = -\frac{1}{2}$

Interim (correlated) rationalizability (Fudenberg, Dekel, Morris 2006)

- Different types have to have the same beliefs in this ex ante view of the world.

Treat each type and nature as a different player.

- interim correlated rationalizability

For each t_i,

$$s_i^0(t_i) = A_i$$

$$a_i \in S_i^k(t_i) \text{ iff } a_i \in \underset{a_i}{\arg\max} \sum_{(\theta, t_{-i}, a_{-i})} u_i(\theta, a_i, a_{-i}) \mu(\theta, t_{-i}, a_{-i})$$

for some μ s.t.

1] $\mu(\theta, t_{-i}, a_{-i}) > 0 \Rightarrow a_{-i} \in S_{-i}^{k-1}(t_{-i})$

2] $p_i(\theta, t_{-i} | t_i) = \sum_{a_{-i}} \mu(\theta, t_{-i}, a_{-i})$

$$S_i^\infty(t_i) = \bigcap_{k=0}^{\infty} S_i^k(t_i)$$

Interim rationalizability is less restrictive than ex ante rationalizability. (Restricting attention to positive probability types)