

I Cooperation in finitely repeated prisoner's dilemma.

a) The case of full information

b) Models with types/reputations

(Gang of 4 approach)

◦ Kreps, Milgrom, Roberts, Wilson

◦ can sustain cooperation (perhaps you must sustain cooperation)

a) Full info: assume $\delta=1$

$t \in \{1, \dots, T\}$

$i \in \{1, 2\}$

$A_t \in \{C, D\}$

	C	D
C	1,1	b/a
D	a/b	0,0

$a > 1$
 $b < 0$

Question 1: $T=1 \Rightarrow$ what are rationalizable strategies?

◦ By iterated strict dominance, (D, D) is unique rationalizable strategy

Question 2: $T > 1$

◦ all stages are reached

◦ consider s_i a Nash equilibrium.

take any h^T , a terminal history.

◦ what is $s_i(h^T)$? if h^T is reached on the eq. path.

Conjecture: All Nash equilibria have same outcome. \square

Shift to sequential rationality.

• For any s_i sequentially rational, $\forall h^T$, $s_i(h^T) = D$

Pf of conjecture: Let s_i^* be a NE in which

$\exists h^T$ terminal on the equilibrium path and $s_i^*(h^T) = C$.

Let $s_i(h) = \begin{cases} s_i^*(h) & \text{if } h \neq h^T \\ D & \text{if } h = h^T \end{cases}$

Profitable deviation. $\rightarrow \leftarrow \square$

Thus, Nash is outcome equivalent to "Defect always".

Ways out:

• Instead of Nash: ϵ -Nash, $\delta < 1$, T large

• (s_i, s_{-i}) ϵ -equilibrium iff $\forall s_i'$, $|u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})| < \epsilon$.

• grim trigger: C if (C,C) before. D otherwise.

• For $\epsilon > 0$, $\delta > 1/2$, $\exists T$ large enough such that grim trigger is ϵ -Nash.

Kreps-Milgrom-Roberts-Wilson:

- Player 1 is rational
- Player 2 is $\left\{ \begin{array}{l} \text{rational} \quad \text{w/pr. } 1-p \\ \text{behavioral} \\ \text{plays grim trigger w/pr } p \end{array} \right.$

ie payoffs:

u_2 w/pr $1-p$

$\left\{ \begin{array}{l} u_2 \quad \text{if play grim trigger} \\ -\infty \quad \text{if not play grim trigger} \end{array} \right.$

We hope to show that for any SPE, there will be cooperation for at least $T-N$ periods, where N does not depend on T . (but does depend on p, b , and a)

Step 1: IF CK that player 2 is rational, then play D every subsequent period. (FI game)

(*) What if w/prob. p , player 2 believes that game is infinitely repeated.

Step 2: If player 1 defects at time t , then independently of his type, player 2 defects every future period

- If grim trigger \rightarrow no choice
- If rational, then if cooperate, I know that 2 is rational, so defect every period in continuation game.
- cannot be optimal to play C since not rewarded for it.

Step 3: at any time t such that player 2's types are pooled $\rightarrow \mu(2 \text{ is behavioral}) = p$

$$\circ V_{\text{coop}}^1 \geq p(T-t)1 - (1-p)b$$

$$\circ V_{\text{defect}}^1 \leq a$$

If both coop before, P_1 cooperates at least until $t = (T - \frac{a+b(1-p)}{p})$

$$\circ \text{let } N_1 = \frac{a+b}{p}$$

Step 4: at $t \leq T - N_1$ and cooperate before,

$$V_{\text{coop}}^2 \geq (T - N_1 - t)$$

$$V_{\text{def}}^2 \leq a$$

2 will cooperate at least until $t = T - N_1 - a$.

$$\text{Let } N = N_1 + a \quad \square$$

$$= \frac{a+b}{p} + a$$