Signaling in Bargaining (Admati, Perry)

- B, S
- S owns an object
- Value to S is 0, to B is l w/pr $1-\pi^0$, h w/pr π^0
- If trade at P and t,
  - S gets $\delta^t P$
  - B gets $\delta^t(b-P)$
- If reject, can delay

Complete information:

- immediate agreement
- equilibrium offer is $P_b = \frac{b}{1+\delta}$
- Buyer offers δP_b

Incomplete info:

- Seq. eq. with intuitive criterion (σ, π)
- History $H^N = (P^0, \Delta^0, P^1, \Delta^1, \ldots, P^N, \Delta^N)$
- $\pi(H^N) = Pr(b=h \mid H^N)$
- $U_b^*(\sigma, H^N)$ - continuation payoff at b
- $V_b^*(H^N)$ - best seq. eq.

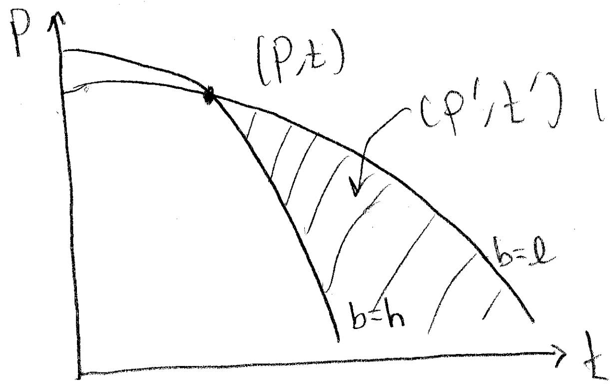
assume: If $\exists$ $H^N, P^{N+1}, \Delta^{N+1}$ s.t.

- $U_b^*(\sigma, H^N) \leq V_b^*(H^N, P^{N+1}, \Delta^{N+1})$
- $U_{b'}^*(\sigma, H^N) > V_{b'}^*(H^N, P^{N+1}, \Delta^{N+1})$
- b' does not want to deviate

If seller sees this, puts prob 0 on b'

In any sequential equilibrium,

1. S never accepts $P < \delta P_\ell$
2. S always accepts $P \geq \delta P_h$
3. B never accepts $P > P_h$
4. B always accepts $P \leq P_\ell$
5. h always accepts $P \leq \underline{P}$ where $h - \underline{P} = \delta(h - \delta P_\ell)$
6. If (P,t) and (P',t') are equilibrium outcomes for h and ℓ, then $t \leq t'$ and $P \geq P'$



7. acceptance occurs with no delay.

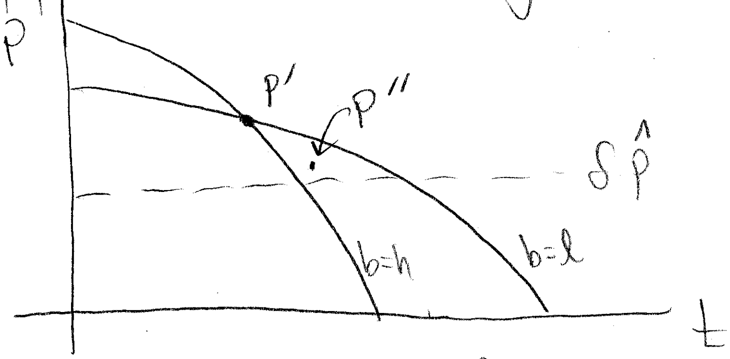
<u>Crucial Lemma</u>

Consider history H^N ending w/ offer P by B. Then

- If $\pi(H^N) = 0$, S accepts P iff $P \geq \delta P_\ell$
- If $\pi(H^N) = 1$, S accepts P iff $P \geq \delta P_h$

<u>Pf:</u> Rejection leads to P_ℓ (this is what we want to show.

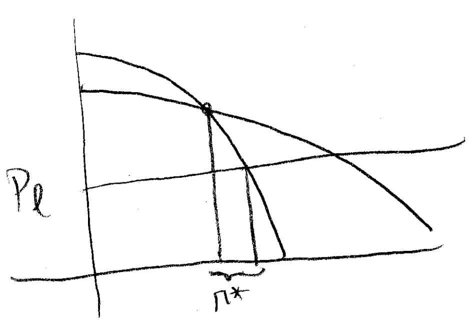
- Let $\hat{p} = \sup\{p \mid (p,t)$ is a SE outcome for $b=l$ after an offer of B is rejected and $\pi=0\}$
- Suppose $\hat{p} > p_l$ to get a contradiction.



- If B counteroffers p'' at t', by A2, then $\pi=0$. l does not make that offer, so S rejects it, because he knows that it is an l type.
- Then $\exists p$ after rejection of B with $\delta p \geq p'' > \delta \hat{p}$ $\Rightarrow p > \hat{p}$, which was a supremum. $\rightarrow\leftarrow$.

Main result.

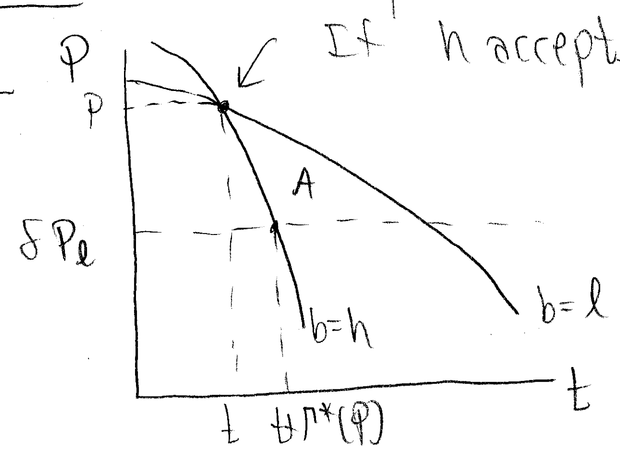
- $\Gamma^*(p)$ = minimum time needed to get signal $b=l$ when h gets p: (ie after $\Gamma^*(p)$, l can credibly signal)
  - $h - p = \delta^{\Gamma^*(p)}(h - \delta p_l)$



__Thm:__ Let P be the first offer made by S at t. Then,

- $P \geq P_l$
- If $P = P_l$, offer is immediately accepted at time $t+1$
- If $P > P_l$,
  - If h accepts P, l counteroffers δP_l at $t + \Gamma^*(P)$, which is accepted
  - If h rejects then counteroffers δP_h at $t+1$, l counteroffers δP_l at $t+1+\Gamma^*(\delta P_h)$.
  - S accepts either offer.

__Pf:__



- Want to show anything in A° (interior) will be accepted
  - By intuitive criterion, S puts prob. 0 on high type.
  - By single crossing property, l will wait until $t+\Gamma^*(p)$ and offer δP_l.

Equilibrium:

- If $\pi^0 \geq \pi^*$, $\exists$ SE with S offering P_h at 0; h accepts; l counteroffers P_ℓ at $\Gamma^*(P_h)$, which is accepted. No such SE otherwise
  - i.e. screening.
- If $\frac{\ell}{h} \geq \pi^0$, $\exists$ a SE where S offers P_ℓ at 0; both types accept. No such SE otherwise.
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