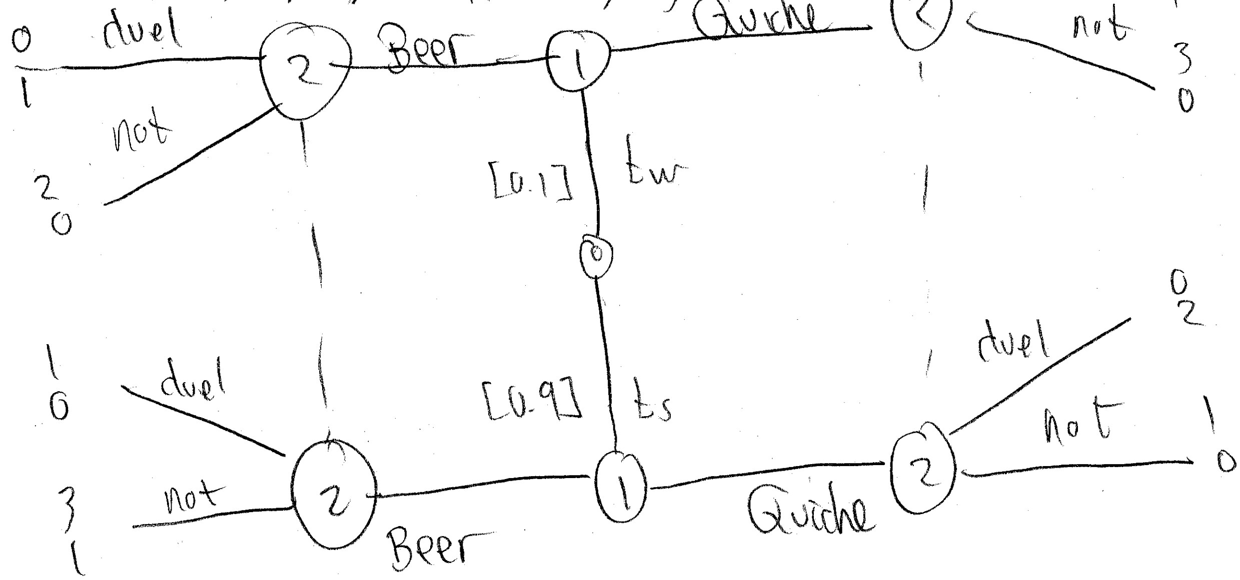
Signaling Games

- Two players: S, R
- Nature selects type $t \in T$ w/ prob $p(t)$
- Sender observes t, chooses $m \in M$
- Receiver observes m, (not t) and chooses action a.
- $U_S(t,m,a)$, $U_R(t,m,a)$



1] Pooling equilibrium: all types choose same message

2] Separating equilibrium: each type chooses a different message

3] Partially separating / pooling

Here: pooling: both players play beer, 2 plays duel if observe beer, not if observe quiche believes $Pr[t_w | \text{Quiche}]$ sufficiently high.

- There is another equilibrium in which both types choose quiche
  - This equilibrium does not make any sense

<u>Forward induction:</u> refinement notion based on assuming that players act rationally.

Suppose t_w is indifferent. This implies that t_s strictly prefers beer.

Suppose t_s is indifferent. Then t_w strictly prefers quiche. This cannot possibly be an equilibrium.

<u>Intuitive criterion</u>

- Fix an equilibrium
- For each t, let $u^*(t)$ be t's eq. payoff.
- For each m off the equilibrium path, define

$$\tilde{T}(m) = \{t \mid u^*(t) > \max_{a \in BR(T(m),m)} u^s(t,m,a)\}$$

- If $\exists (t', m)$ s.t.

$$u^*(t') < \min_{a \in BR(T(m) \setminus \tilde{T}(m), m)} u^s(t', m, a)$$

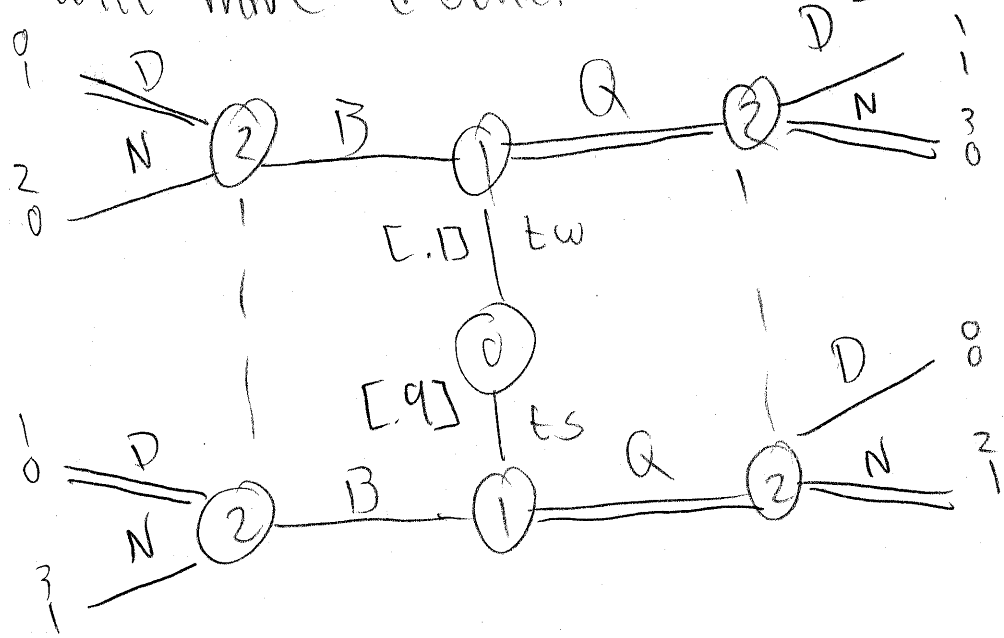
Then the equilibrium fails the intuitive criterion.

- player two has a belief $\mu \in \Delta(T)$
  - will play a BR to μ, given sender has chosen some message m.
- $T(m) = \{t \mid m$ is available for $t\}$

- $BR(T(m), m) = \bigcup_{\mu \in \Delta(T(m))} BR_2(\mu, m)$

$\Rightarrow \tilde{T}(m) = \{t \mid t$ never wants to play $m\}$

- If 2 sees m, he knows $t \notin \tilde{T}(m)$. He will have a belief over $\Delta[T(m) \setminus \tilde{T}(m)]$



- $u^*(t_w) = 3$, $u^*(t_s) = 2$

$\max_a u(t_w, beer, a) = 2 < u^*(t_w)$

$\max_a u(t_s, beer, a) = 3 > u^*(t_s)$

$\Rightarrow \tilde{T}(\text{beer}) = \{t_w\}$

$\Rightarrow \mu(t_s | \text{beer}) = 1$

$\Rightarrow$ Not fight

$\Rightarrow$ t_s will drink beer

- Thus, this fails the intuitive criterion.

Testing an equilibrium

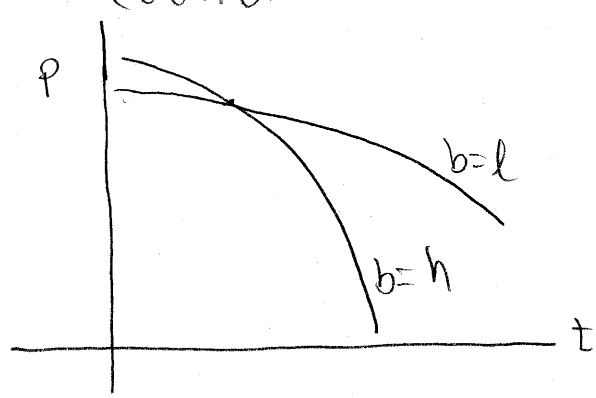
- $U^*(t)$ = expected payoff of type t in equilibrium
- Pick criterion: a will not be taken if $a \notin BR(T(m), m)$. Iterate $T^S(m)$

Hmm...

Signaling in Bargaining Incomplete information plays a big role.

- try to signal your valuation as a buyer is low.
- seller can try to screen: set high price initially. Set low price later
- Both are important

- Buyer, Seller
- Seller owns object. Value is:
  - 0 to seller
  - $b = l, h$ where $h > l > 0$. $Pr[b=h] = \pi^0$
- If trade at price P at time t
  - seller gets $\delta^t P$
  - Buyer gets $\delta^t (b-P)$
- alternating offers.
  - seller makes first offer
  - afterwards, responder chooses to wait $\Delta \geq 1$ and either accepts or makes counter offer.



$\delta^t (h-P) = k$

$\Rightarrow P = h - k\left(\frac{1}{\delta}\right)^t$

$\delta^t (l-P) = k$

$\Rightarrow P = h - k\left(\frac{1}{\delta}\right)^t$

If complete info:

Equilibrium offer: $P_b = \frac{b}{1+\delta}$

- will choose Δ as small as possible.

Buyer offers δP_b

Sequential equilibrium with intuitive criterion (σ, π)

- assume: if can obtain same payoff with smaller offers, make smaller offer
- History $H^N = (p^0, \Delta^0, p^1, \Delta^1, \ldots)$
- $\pi(H^N) = Pr[b = h | H^N]$
- $U_b^*(\sigma, H^N)$ = continuation payoff of b
- $V_b^*(H^N)$ best sequential equilibrium outcome for b after H^N

assume: If

- $U_b^*(\sigma, H^N) \leq V_b^*(H^N, p^{N+1}, \Delta^{N+1})$
- $U_{b'}^*(\sigma, H^N) > V_{b'}^*(H^N, p^{N+1}, \Delta^{N+1})$
  - intuitive criterion puts probability zero on b' after $(H^N, p^{N+1}, \Delta^{N+1})$