$\boxed{\text{I}}$ Proof by induction carefully
$\boxed{\text{II}}$ Details on problem 3
$\boxed{\text{III}}$ Cooperation in finitely repeated PD.

$\boxed{\text{I}}$ Proof by induction
$k \in \mathbb{N} \setminus \{0\}$
$\mathcal{H}_k$ - induction hypothesis at stage k
∘ prove that $\mathcal{H}_1$ holds and that $\mathcal{H}_k \Rightarrow \mathcal{H}_{k+1}$.

Question 1a

We proceed by induction on the stage of the game G.
Let k be the # of non-terminal histories
Denote $|G| = k$
$\mathcal{H}_k$ = any game with $|G| \leq k$ has a unique strategy that survives iterated elimination of conditionally dominated strategies

To show $\mathcal{H}_1$ holds:
∘ if $|G| = 1$, then it is a 1 person decision problem.
∘ u 1-1 implies $\exists!$ u-maximizing action
$\Rightarrow \mathcal{H}_1$ holds

To show $H_k \Rightarrow H_{k+1}$:

- Consider $|G|$ s.t. $|G| = k+1$. Take any SPE of G
- pick h a final decision node
- at h, $i(h)$, $s_{i(h)}(h)$ maximizes $i(h)$'s utility given h
  - 1-1 $\Rightarrow \exists!$ action $s_{i(h)}(h)$ consistent with SPE

$\Rightarrow s = (s_i, s_{-i})$ must be SPE of game G' where h is turned into final node with payoffs $u_j(s_{i(h)}(h) \mid h)$

- $|G'| = k$ $\square$

Steps: 1] choose something you want to induct on: k

2] Define H_k

3] Prove H_1

4] Prove $H_k \Rightarrow H_{k+1}$

E-mail Game by Rubinstein has a good proof by induction in a repeated game.

Purification results

- mixed eq. in game G very close to pure eq. in game $\hat{G}$, an elaboration of G

eg: at each decision node, get signal $u \in [0,1]$

- $\tilde{\pi} = \pi + 0 \cdot u$ and then action is a function of u.

Problem 3: $g(u_2) = \max\{u_1 : (u_1, u_2) \in \mathbb{X}\}$

a) From class, $\exists!$ SPE where

1 proposes $(x_1, g^{-1}(x_1))$

1 accepts $(g(x_2), x_2)$

where $x_1 = g(\delta_2 g^{-1}(\delta_1 x_1))$

here $g(x_2) = 1 - x_2$

Let $f(x_1) = g(\delta_2 g^{-1}(\delta_1 x_1))$

(*) More precisely,

$$x_1 = 1 - \delta_2(1 - \delta_1 x_1)$$
$$= 1 - \delta_2 + \delta_1 \delta_2 x_1$$
$$\Rightarrow x_1(1 - \delta_1\delta_2) = 1 - \delta_2$$
$$\Rightarrow x_1 = \frac{1-\delta_2}{1-\delta_1\delta_2}$$
$$x_2 = \frac{1-\delta_1}{1-\delta_1\delta_2}$$

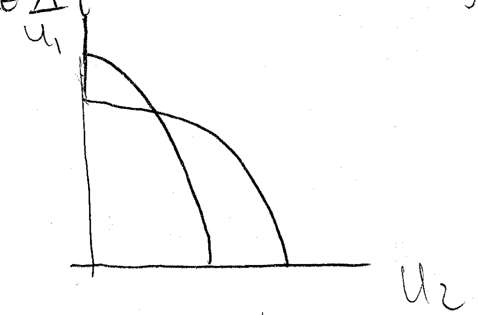
Note that $x_2 \neq 1 - x_1$

b) Continuity of $x_1(\delta_1, \delta_2)$ as $(\delta_1, \delta_2) \to (1,1)$

(Can you extend $x_1(\cdot,\cdot)$?)

(b) $\bar{X}_1 \neq \bar{X}_2$

- $\max\limits_{x \in \bar{X}_i} x_i < \max\limits_{x \in \bar{X}_j} x_i \quad \forall i \neq j$



- could have had $\max\limits_{x \in \bar{X}_i} x_i > \max\limits_{x \in \bar{X}_j} x_i \quad \forall i \neq j$

- $\bar{v}_i = \max\limits_{\{s \text{ survives ICD}\}} u_i(s)$ does this necessarily exist?
- $\underline{v}_i = \inf\limits_{\{s_j \text{ survives ICD}\}} \sup\limits_{\{s_i \text{ survives ICD}\}} u_i(s)$

This is ex ante payoffs when 1 starts proposing.

- Conceptually, $\bar{v}_1 \neq \bar{v}_2$, $\underline{v}_1 \neq \underline{v}_2$

Candidate

Right approach (proof strategy)
(b) + (c) together

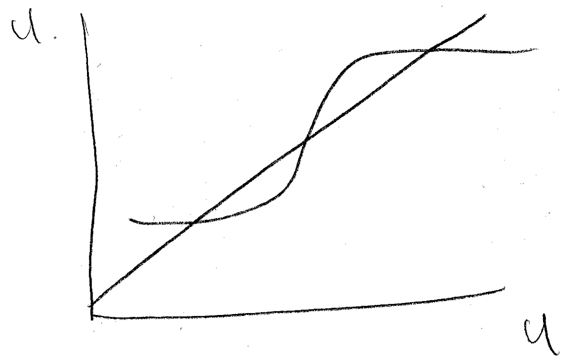
Define: $u_1^{l,0} = 0$, $u_2^{l,0} = 0$

$u_1^{h,0} = \max\limits_{x \in \bar{X}_1} x_1 \quad u_2^{h,0} = \max\limits_{x \in \bar{X}_2} x_2$

- $g_1(u_2) = \max\{u_1 \mid (u_1, u_2) \in \bar{X}_1\}$
- $g_2(u_1) = \max\{u_2 \mid (u_1, u_2) \in \bar{X}_2\}$

Then $u_1^{l,1} = g_1(\delta_2 u_2^{h,0})$, which exists by the weird assumption.

<u>In the end</u>, we get $u_1^{l,\infty}, u_2^{l,\infty}, u_1^{h,\infty}, u_2^{h,\infty}$



$f_\varepsilon \to f$ uniformly over K as $\varepsilon \to 0$

Q^0 = when fp of f_q close to f.p. of f

When do extreme fixed points of f^{δ_1, δ_2} converge to extreme fixed points of $f^{1,1}$?