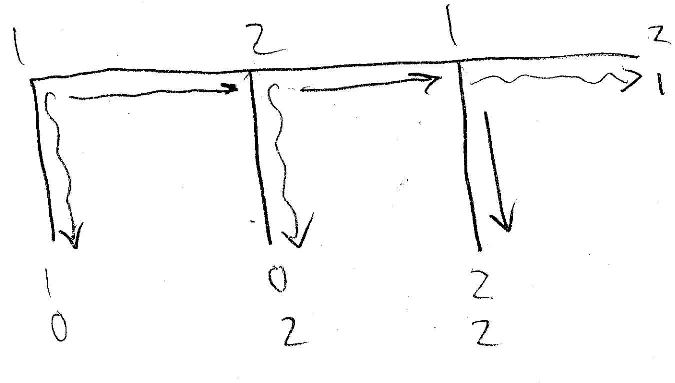
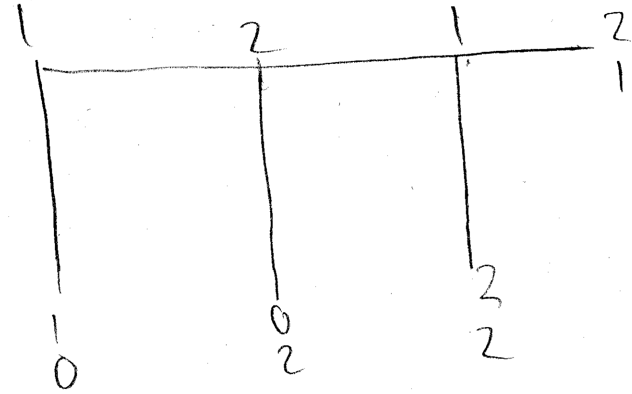
Backward induction vs. conditional dominance with non-generic payoffs:

- Two backward induction equilibria. (pure)
- Nothing is conditionally dominated.

Generically the two concepts coincide, though (ie w/no ties.)

Solving bargaining by backward induction yields unique predictions. (immediate agreement)

- solving by iterated conditional dominance, we do not have such sharp predictions.

	A	B
A	1,1	0,0
B	0,0	0,0

Both (A, A) and (B, B) (along with an appropriate belief system) are a sequential equilibrium.

(B, B) is not trembling hand perfect.

THP.

- consider normal form games

Defn: σ is a THP equilibrium iff $\exists$ a sequence of completely mixed strategy profiles σ^m s.t.

- $\sigma^m \to \sigma$
- σ_i is a best reply to σ_{-i}^m $\forall m, i$.

e.g.

	L	R
X	2,2	2,2
T	1,0	3,1
B	1,0	0,1

- Verify that (T, R) is a THP eq.

Let $\sigma_1^n = (\frac{1}{n}, 1-\frac{2}{n}, \frac{1}{n})$ Then

$u_2(R, \sigma_1^m) = \frac{2}{n} + 1 - \frac{2}{n} + \frac{1}{n} = 1 + \frac{1}{n}$

$u_2(L, \sigma_1^m) = \frac{2}{n}$

- For $n > 2$, R is a BR to σ_1^n $\forall n$.

It is easy to show that T is a BR for 1 to $\sigma_2^n = (\frac{1}{n}, 1-\frac{1}{n})$.

3,1	0,0	3/4
0,0	1,3	1/4
1/4	3/4	

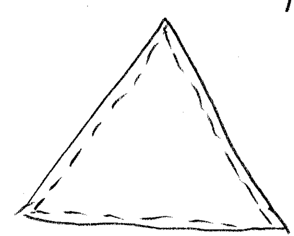
- 2 pure, 1 mixed
- any strict NE is THP
- any completely mixed NE is also THP, since can take trivial mixture.

<u>Equivalent definition</u>

an <u>ε-constrained eq.</u> is a NE of a game where each s_i has at least prob. $\varepsilon(s_i)$. a

<u>THP</u> is a limit of ε-constrained eq. as $\varepsilon \to 0$.

- ie we restrict strategies to interior of mixed strat. simplex



<u>Existence</u>

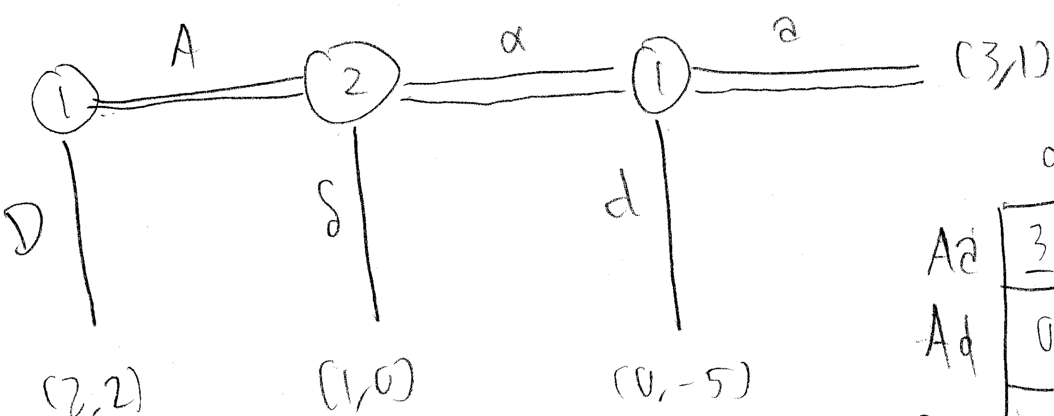
<u>Thm:</u> Every finite game $G=(N,S,u)$ has a THP

<u>Pf:</u> Define $G^\varepsilon = (N, S, u^\varepsilon)$ by

$$u^\varepsilon(s) = (1-\varepsilon)u(s) + \varepsilon \sum_{s' \in S} \frac{u(s')}{|S|}$$

- utility vector

- G^ε has a NE σ^ε, which is $(\varepsilon/|S|)$-constrained equilibrium in G.
- $\sigma^* = \lim_{\varepsilon \to 0} \sigma^\varepsilon$ is a THP.



	α	δ
Aa	3,1	1,0
Ad	0,-5	1,0
Da	2,2	2,2
Dd	2,2	2,2

- convert to normal form:
- Aa is a strict NE ⇒ Aa is THP
- consider perturbation that puts lots of weight on Ad. Then δ is BR.
  - similarly for player 1
  - (Da, δ), (Dd, δ) are THP
- THP are NE that are not weakly dominated in two-player game.

Agent normal form:

A

	α	δ
a	$\boxed{3,1}$	1,0
d	~~0,-5~~	~~1,0~~

~~D~~

	α	δ
a	2,2	2,2
d	~~2,2~~	~~2,0~~

- a weakly dominated strategy could never be a BR to a completely mixed strategy

- The THP of an agent normal form game is a sequential equilibrium.

Thm: Every THP of agent normal form game is sequential. Converse is also true for generic payoff fcns but not all.

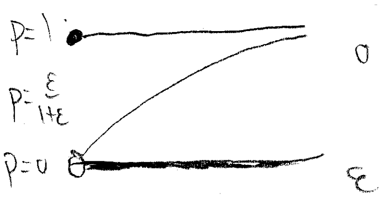
Question: Is THP uhc?

- No.

1,1	0,0
0,0	0,0

consider $\Longrightarrow$

p	1,1	0,0
	0,0	ε,ε



- when $\varepsilon=0$, we have unique THP
- when $\varepsilon>0$, we have three THP.
  - we really need to know that $\varepsilon=0$.

Sequential equilibrium is upper-hemicontinuous

Generically, THP is equiv. to sequential eq. and is hence uhc.