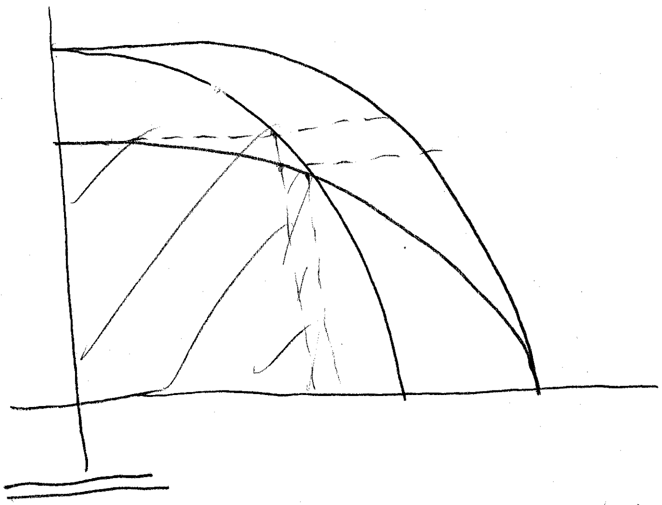


Iterated conditional dominance in bargaining

- $g_2 = g;\ g_1 = g^{-1}$ $\quad g_2(v_1) =$ max 2 could get if 1 is getting v_1.
- $f_i(x_i) = g_i(\delta_j g_j(\delta_i x_i))$ $\quad g_1(v_2) =$ max 1 could get if 2 is getting v_2
  - increasing.

- $v_i^m = \inf\limits_{\{s(j) \text{ survives rd. } m\}} \ \sup\limits_{s(i)} E[u_i(s) \mid i \text{ offers}]$

- $V_i^m = \sup\limits_{\{s \text{ survives rd } m\}} E[u_i(s) \mid i \text{ offers}]$

- $v_i^m \le x_i^* \le V_i^m$, where x_i^* is i's share in his SPE offer

Round m+1:

- Reject x with $x_i > \delta_i V_i^m$ dominated
- accept x with $x_i < \delta_i v_i^m$ dominated
  - if reject, get $\delta x_i^* \ge \delta v_i^m$
- $v_j^{m+1} \ge g_j(\delta_i V_i^m)$ and $V_j^{m+1} \le g_j(\delta_i v_i^m)$

$\Rightarrow v_i^{m+2} \geq f_i(v_i^m)$, $V_i^{m+2} \leq f_i(V_i^m)$

- Since $V_j^{m+1} \leq g_i(\delta_i v_i^m)$,

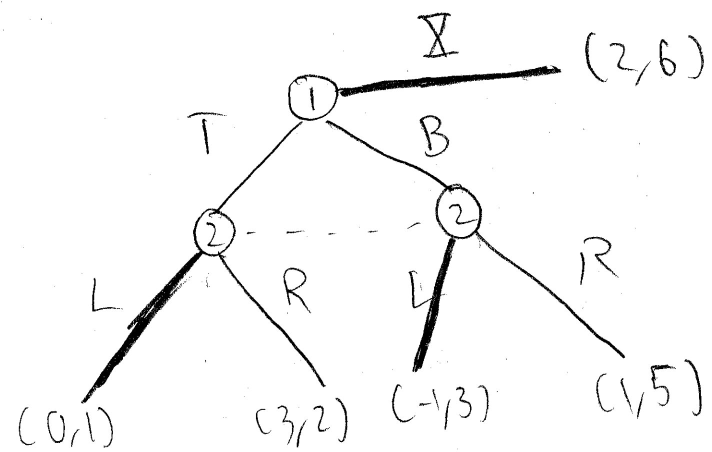
$$v_i^{m+2} \geq g_i(\delta_j V_j^{m+1}) \geq g_i(\delta_j g_j(\delta_i v_i^m)) = f_i(v_i^m)$$

- $v_i^\infty \geq \min_{\text{fixed pt of } f} f$. Similarly $V_i^\infty \geq \max_{\text{fixed pt of } f} f$

Then, since $f_i(\cdot)$ has a unique fixed point,

$$\lim_{m\to\infty} v_i^m = x_i^* = \lim_{m\to\infty} V_i^m$$

Equilibrium Refinements

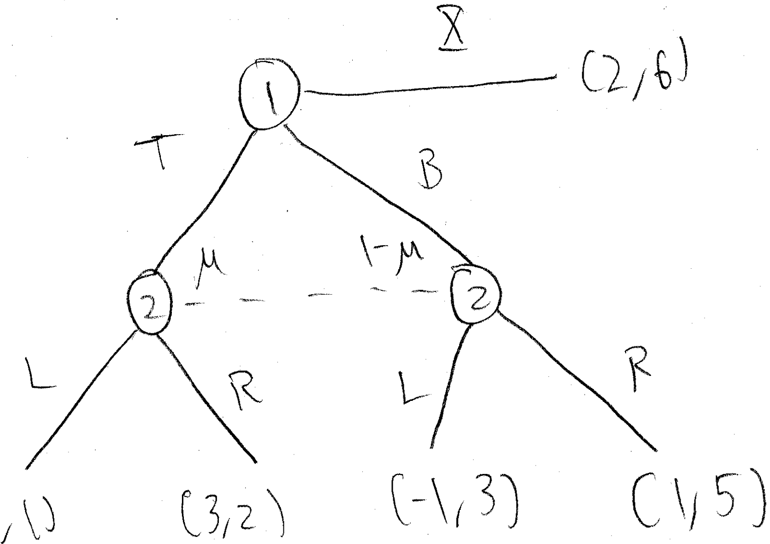


- This is a SPNE
- since there are no subgames.

Selten: ② should play R.

- actions should be best reply to any small trembles of the other player.
- Trembling hand perfect equilibrium

Kreps and Wilson (1982) - Sequential equilibrium
- player two must have some belief about which node in the information set he is at.



- For any $\mu \in [0,1]$, player two should play R.

a player is sequentially rational at a history h if he plays a BR to a belief conditional on being at that history.

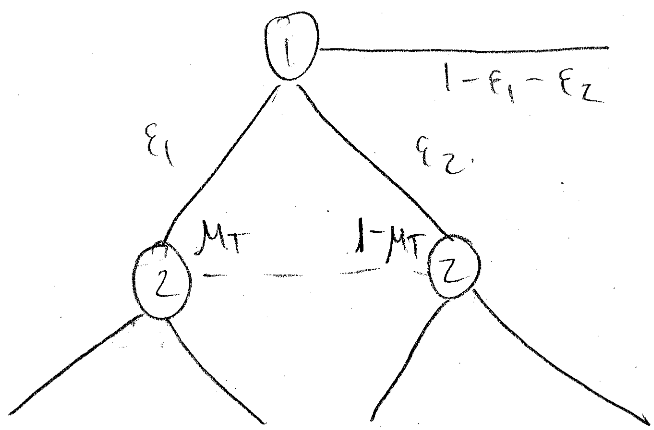
Sequential Equilibrium

an assessment: (σ, μ) where σ is a strategy profile and μ is a belief system, $\mu(h) \in \Delta(h)$ for each h.

an assessment (σ, μ) is <u>sequentially rational</u> if at each h_i, σ_i is a BR to σ_{-i} given $\mu_i(h_i)$.

- $\forall i, h_i, \sigma_i'$, $E_{\mu_i(h_i)}[u_i(\sigma_i, \sigma_{-i}|h_i] \geq E_{\mu_i(h_i)}[u_i(\sigma_i', \sigma_{-i})|h_i]$

(σ, μ) is <u>consistent</u> if $\exists$ a sequence (σ^m, μ^m) $\to (\sigma, \mu)$ where σ^m is completely mixed (every history has positive probability) and μ^m is computed from σ^m by Bayes rule:

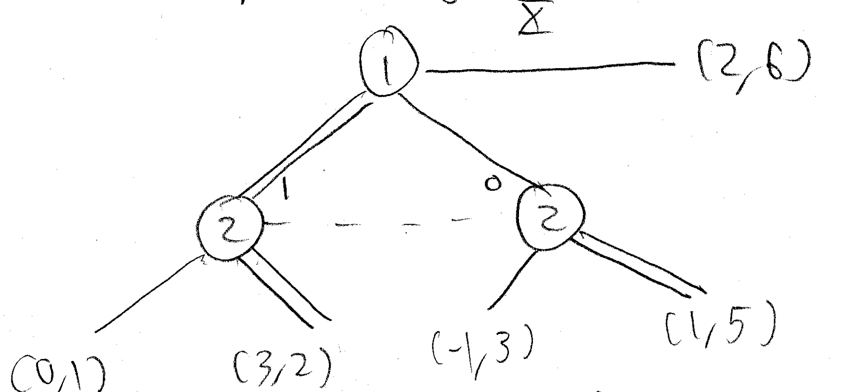


- $\mu(T|h) = \frac{Pr[T; \sigma^\varepsilon]}{Pr[h; \sigma^\varepsilon]}$

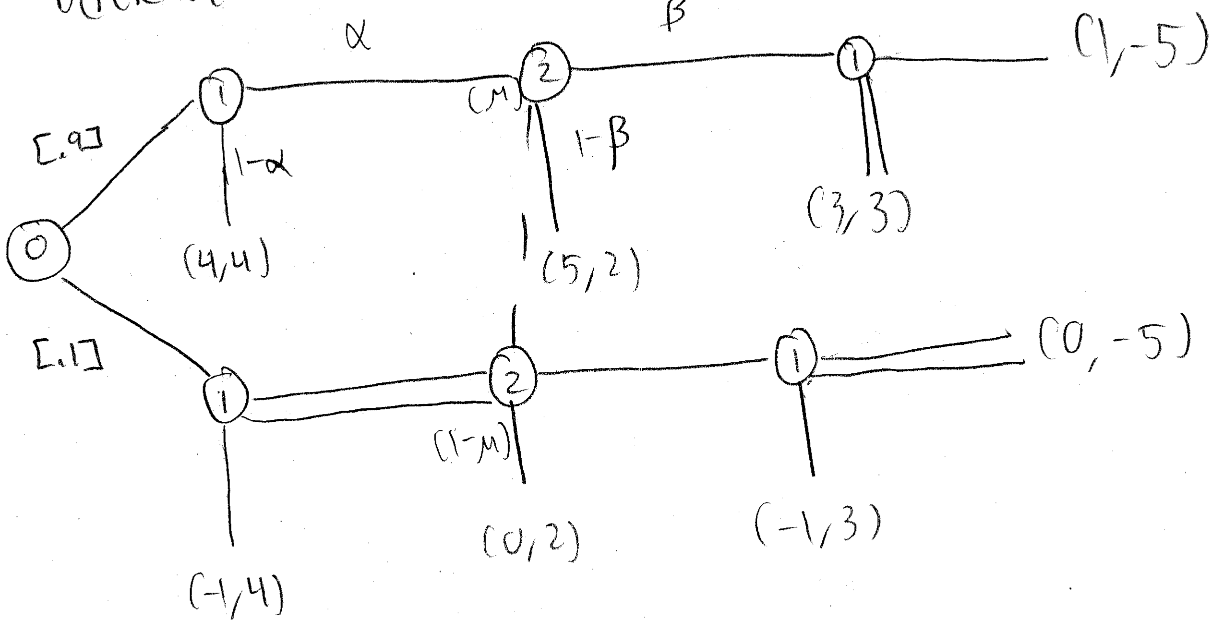
$= \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}$

- on the path, there is one belief. Off the path, there are a lot of beliefs.
- σ^m is completely mixed
  - $\forall h$: $Pr(h|\sigma^m) = \sum_{\{s: \text{reaches } h\}} \sigma(s) > 0$
  - $\mu^m(x|h) = \frac{\sum_{\{s: \text{reaches } x\}} \sigma(s)}{\sum_{\{s: \text{reaches } h\}} \sigma(s)}$

an assessment (σ, μ) is a <u>sequential equilibrium</u> if it is sequentially rational and consistent.



With perfect information this coincides with backwards induction.



$$\mu = \frac{0.9\alpha}{0.9\alpha + 0.1(1)} = \frac{.9\alpha}{.9\alpha + .1}$$ consistency

(SRP1): $\alpha = \begin{cases} 1 & \beta < \frac{1}{2} \\ [0,1] & \beta = \frac{1}{2} \\ 0 & \beta > \frac{1}{2} \end{cases}$

(SRP2):

$$\beta = \begin{cases} 1 & \mu > \frac{7}{8} \\ [0,1] & \mu = \frac{7}{8} \\ 0 & \mu < \frac{7}{8} \end{cases}$$

$\alpha = 1 \Rightarrow \mu = 0.9 > \frac{7}{8} \Rightarrow \beta = 1 \Rightarrow \alpha = 0 \quad \rightarrow\leftarrow$

$\alpha = 0 \Rightarrow \mu = 0 < \frac{7}{8} \Rightarrow \beta = 0 \Rightarrow \alpha = 1 \quad \rightarrow\leftarrow$

$\alpha \in (0,1) \Rightarrow \beta = \frac{1}{2} \Rightarrow \alpha^* = \frac{7}{9}$

s.t. $\alpha = \alpha^*$

Properties of sequential eq.

1] In finite games, sequential eq.

a] exists

b] is uhc

c] has finitely many SE-outcomes for generic payoffs

d] Refines SPE

2] If game is continuous at ∞, single-deviation principle applies