I Single deviation principle

II Bargaining

III Cooperation in finitely repeated PD

Single deviation principle (SDP)

Want to check if (s_i, s_{-i}) is SPE

Conditions
- Game is continuous at infinity.
- Pick history h_0, define $\tilde{s}_i$ s.t. $\forall h \neq h_0$, $\tilde{s}_i(h) = s_i(h)$, but at $h = h_0$, $\tilde{s}_i(h_0) = \tilde{a}_i \neq s_i(h_0)$.

SDP. If $\forall$ such deviations at all histories, not profitable, then (s_i, s_{-i}) is SPE.

(C) (i.e. $\forall i, h_0, \tilde{s}_i$, $u_i(s_i, s_{-i} | h_0) \geq u_i(\tilde{s}_i, s_{-i} | h_0)$, then (s_i, s_{-i}) is SPE)

Simpler to check than full SPE condition

Thm: When a game is continuous at infinity and (s_i, s_{-i}) satisfies (C), then (s_i, s_{-i}) is a SPE

Pf: By contradiction

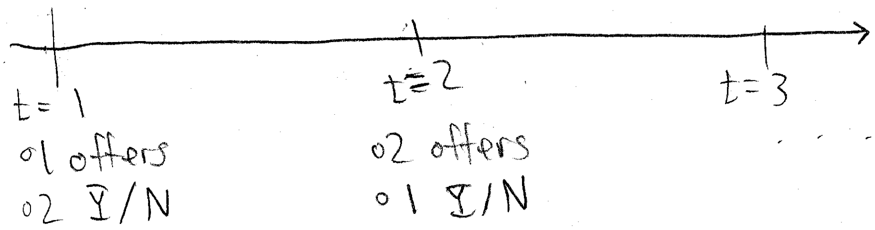
Take (s_i, s_{-i}). Assume there exists $\tilde{s}_i$ which is a profitable deviation at some h_0. Then
- $u_i(\tilde{s}_i, s_{-i} | h_0) > u_i(s_i, s_{-i} | h_0)$

- Because the game is continuous at infinity, there exists a finite deviation s_i' (i.e. $\#\{h \mid s_i'(h) \neq s_i(h)\} = K < +\infty$) that is profitable.
  - (if an infinite pd deviation exists, let s_i' agree until some T, beyond which the deviation contributes at most $\delta^T k$, which is small, by continuity at infinity.)
- Induction on $K = \#$ pds at which s_i' differs from s_i
- pick h^* s.t. $s_i'(h^*) \neq s_i(h^*)$ but s_i' and s_i coincide afterwards.
- Then, note that, conditional on h^*, s_i' is a single period deviation from s_i
  - by (c), $u_i(s_i, s_{-i} \mid h^*) \geq u_i(s_i', s_{-i} \mid h^*)$
  - $\Rightarrow$ can improve s_i' at h^* to $s_i''(h^*) = s_i(h^*)$
  - if there is a profitable deviation that differs at K nodes, then $\exists$ a profitable deviation that differs at $K-1$ nodes. □

$\boxed{\text{II}}$ Bargaining model Muhamet went through (Rubinstein)

- $g(u_2) = \max\{u_1 : (u_1, u_2) \in X\}$

- $g^{-1}(u_1) = \max \{u_2 : (u_1, u_2) \in \mathbb{X}\}$



Clearly $u_1 \geq \underline{u}_1^0 = 0$ and $u_1 \leq \bar{u}_1^0 = g(0)$
$u_2 \geq \underline{u}_2^0 = 0$ $u_2 \leq \bar{u}_2^0 = g^{-1}(0)$

Given that 2 cannot hope for more than $\bar{u}_2^0$ next pd if he rejects, 1 knows that any offer s.t. $u_2 \geq \delta \bar{u}_2^0$ gets accepted. Thus, player 1 can expect to get at least $u_1 \geq \underline{u}_1^1 = g(\delta \bar{u}_2^0)$

Similarly, $u_2 \geq \underline{u}_2^1 = g^{-1}(\delta \bar{u}_1^0)$

Now, 1 knows that 2 can guarantee $\underline{u}_2^1 = g^{-1}(\delta \bar{u}_1^0)$ by refusing, this gives us an upper bd to what 1 can hope to get:

$\bar{u}_1^1 = g(\delta \underline{u}_2^1) \geq u_1$

Similarly, $u_2 \leq \bar{u}_2^1 = g^{-1}(\delta \underline{u}_1^1)$

Now, define the sequences $\bar{u}_1^k, \underline{u}_1^k$, $\bar{u}_2^k, \underline{u}_2^k$ as follows:

$\underline{u}_1^{k+1} = g(\delta \bar{u}_2^k)$, $\underline{u}_2^{k+1} = g^{-1}(\delta \bar{u}_1^k)$

$\bar{u}_1^{k+1} = g(\delta \underline{u}_2^{k+1})$, $\bar{u}_2^{k+1} = g^{-1}(\delta \underline{u}_1^{k+1})$

Lemma: If u_1, u_2 are such that $\forall k, u_i \in [\underline{u}_i^k, \bar{u}_i^k]$

$$\bar{u}_1^{k+1} = g(\delta \underline{u}_2^{k+1}) = g(\delta g^{-1}(\delta \bar{u}_1^k)) \equiv f(\bar{u}_1^k)$$

- composition of two decreasing functions, so f is increasing.

$$\underline{u}_1^{k+1} = f(\underline{u}_1^k)$$

Question: fixed points of f. Want to find some

u_1 s.t. $u_1 = g(\delta g^{-1}(\delta u_1))$

$$\Rightarrow g^{-1}(u_1) = g^{-1}(\delta g^{-1}(\delta u_1)) = \delta g^{-1}(\delta u_1)$$

Call $u_2 = g^{-1}(u_1) \Rightarrow g(u_2) = u_1$

Then $u_2 = \delta g^{-1}(\delta g(u_2))$

$$\Rightarrow g\left(\frac{u_2}{\delta}\right) = \delta g(u_2)$$

Is there an intersection of:

$h_a(u_2) = \delta g(u_2)$

$h_b(u_2) = g(\frac{1}{\delta} u_2)$?