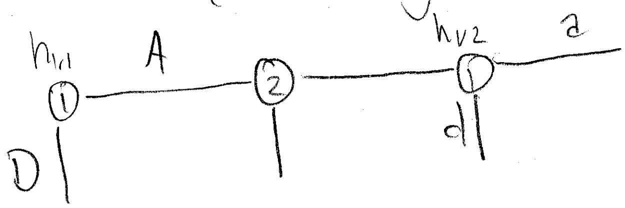
- Mixed strategy: $\sigma_i \in \Delta(S_i)$
  - mixture over contingent plans
- Sometimes, it is more convenient to think about probability distributions over actions at each information set
- Behavioral strategy: $b_i(h_i) \in \Delta(A(h_i))$, $h_i \in H_i$
- b_i induces σ_i with $\sigma_i(s_i) = \Pi[b_i(h_i)](s_i(h_i))$.

$$\begin{array}{l} \sigma \\ 1/2 \ Aa \\ 0 \ Ad \\ 0 \ Da \\ 1/2 \ Dd \end{array} \Bigg\} \Rightarrow \begin{pmatrix} A \ \text{w/pr } 1/2 \\ D \ \text{w/pr } 1/2 \end{pmatrix}, (a) \Rightarrow \left\{ \begin{array}{lll} \overset{\sigma'}{Aa} & \frac{1}{2} \cdot 1 = \frac{1}{2} \\ Ad & \frac{1}{2} \cdot 0 = 0 \\ Da & \frac{1}{2} \cdot 1 = \frac{1}{2} \\ Dd & \frac{1}{2} \cdot 0 = 0 \end{array} \right.$$

$\Rightarrow \sigma, \sigma'$ are equivalent

- $\forall \sigma$, we can pin down a unique b
- $\forall b$, we can pin down an element σ of some equivalence class.

- $R_i(h_i) = \{s_i \mid h_i \text{ is on the path of some } (s_i, s_{-i})\}$
  $= \{\text{strategies that will reach history } h_i\}$



$R_1(h_{1,1}) = \{Aa, Ad, Da, Dd\} = S_1$

$R_1(h_{1,2}) = \{Aa, Ad\}$

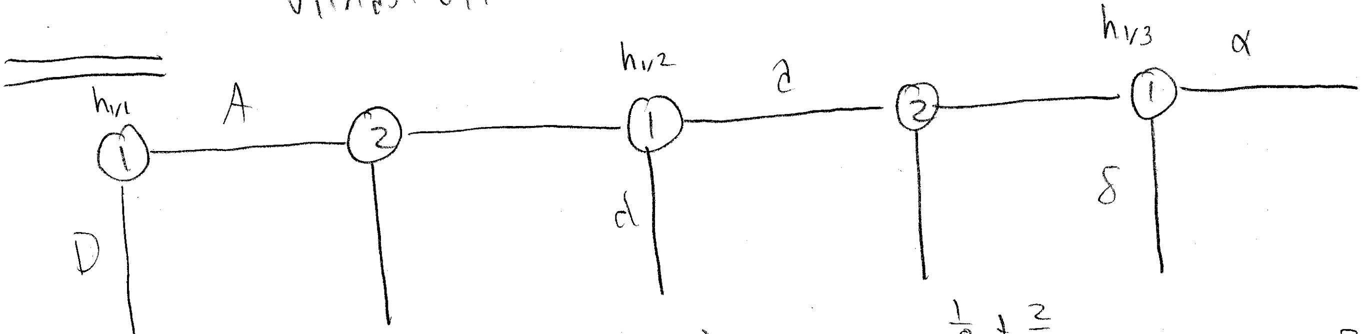
- Suppose $\sigma_i(R_i(h_i))>0 \Rightarrow b_i(a_i|h_i) = \dfrac{\Pr(\text{playing } a_i \text{ and reaching } h_i)}{\Pr(\text{reaching } h_i)}$

$$\Rightarrow b_i(a_i|h_i) = \frac{\sum_{\substack{s_i\in R_i(h_i)\\ s_i(a_i)=a_i}} \sigma_i(s_i)}{\sigma_i(R_i(h_i))} \quad \text{if } \sigma_i(R_i(h_i))>0$$

$$b_i(a_i|h_i) = \begin{cases} \dfrac{\sum_{\substack{s_i\in R_i(h_i)\\ s_i(a_i)=a_i}} \sigma_i(s_i)}{\sum_{s_i\in R_i(h_i)} \sigma_i(s_i)} & \sigma_i(R_i(h_i))>0 \\ \text{"anything"} & \sigma_i(R_i(h_i))=0 \end{cases}$$

Let $\sigma=(\sigma_1(Aa),\sigma_1(Ad),\sigma_1(Da),\sigma_1(Dd))=(\frac{1}{2},0,0,\frac{1}{2})$

$$b_1(a|h_{1,2}) = \frac{\sigma_1(Aa)}{\sigma_1(Aa)+\sigma_1(Ad)} = \frac{\frac{1}{2}}{\frac{1}{2}+0} = 1$$



$$\left.\begin{aligned} \sigma_1(Aa\alpha) &= \tfrac{1}{8} \\ \sigma_1(Aa\delta) &= \tfrac{2}{8} \\ \sigma_1(Ad\delta) &= \tfrac{1}{8} \\ \sigma_1(Da\alpha) &= \tfrac{4}{8} \end{aligned}\right\}$$

$$b(a|h_{1,2}) = \frac{\frac{1}{8}+\frac{2}{3}}{\frac{1}{8}+\frac{2}{8}+\frac{1}{8}} = \frac{3}{4}$$

$$b(\alpha|h_{1,3}) = \frac{\frac{1}{8}}{\frac{1}{8}+\frac{2}{8}} = \frac{1}{3}$$

$$b(A|h_{1,1}) = \frac{\frac{1}{8}+\frac{2}{8}+\frac{1}{8}}{\frac{1}{8}+\frac{2}{8}+\frac{1}{8}+\frac{4}{8}} = \frac{1}{2}$$

Suppose $\sigma_1(D a\alpha)=1$. We will typically write, for off-the-equilibrium path actions, (ie $\sigma_i(R_i(h_i))=0$)

$$b_i(a_i|h_i) = \sum_{\{s_i(h_i)=a_i\}} \sigma_i(s_i)$$

Thm: (Kuhn): Under perfect recall, mixed and behavioral strategies are equivalent

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Solution concepts:

Most know:

- NE (NE of normal form game implied by ext. form)
- SPNE
- Backwards induction

Here, we will learn:

- Iterated conditional dominance
- Sequential equilibrium
- Perfect equilibrium

Repeated Games with Observable actions

- $T = \{1, 2, \ldots, t, \ldots\}$
- G = simultaneous action game (stage game)

e.g.

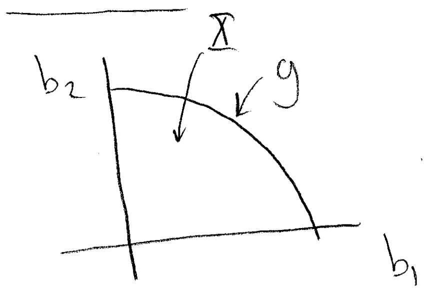
	C	D
C	5,5	-1,6
D	6,-1	1,1

- at each $t \in T$, G is played, and players remember actions taken before t.
- Payoffs: Discounted sum of payoffs in stage game

- Stages $k = 0, 1, 2, \ldots$
- at $k = 0$,
  - Players $N(\varphi)$ are active at stage φ
  - each $i \in N(\varphi)$ plays an action $a_i^0 \in A_i(\varphi)$
- at $k = 1, 2, \ldots, K, \ldots$, and history $h \in \{(a^0, a^1, \ldots, a^{k-1})\}$
  - players $N(h)$ are active
  - history $h \in \{(a^0, a^1, \ldots, a^{k-1})\}$ is commonly known
  - each $i \in N(h)$ plays $a_i^k(h) \in A_i(h)$
- Payoffs: $u(h)$ at each terminal history

Defn: a game of perfect information is one for which at h, only one player is active.

For repeated stage games, subgames begin after every stage.



- $N = \{1, 2\}$
- X = feasible expected utility pairs
  - assume X is convex
- $U_i(x, \underbrace{t}_{\text{decision reached at } t}) = \delta_i^t x_i$
- $D = (0,0) \in X$ - disagreement payoffs
- g is concave, continuous, and strictly decreasing

- $T = \{0, 1, \ldots, t, \ldots\}$
- at $t = 2n$,

- 1 offers x
- 2 accepts or rejects
- If accepted, game ends, and payoffs are x
- otherwise, proceed to $t+1$

- at $t=2n+1$,
  - 2 offers y
  - 1 accepts or rejects
  - If accepted, game ends, yielding y
  - otherwise, proceed to $t+1$