Buyer: $i=1,2$; $v_i \in \{v_L, v_H\}$, $Pr[v=v_H]=\beta$
Seller cost $= 0 < v_L < v_H$
auction defines payment p_{ij} and prob. of sale x_{ij}
if buyer announces v_i and other buyer offers v_j

Feasibility: $2x_{HH} \leq 1$, $2x_{LL} \leq 1$, $x_{LH} + x_{HL} \leq 1$
Efficiency: $x_{HH} = x_{LL} = \frac{1}{2}$, $x_{HL} = 1$, $x_{LH} = 0$
<u>Private</u>, <u>known</u>, <u>independent</u> values with risk neutral buyer

(*) Given that a buyer can always turn around and sell what she just bought, why can values ever be private?

$u = xv - p$

Optimal (revenue maximizing for the seller) auction?
- also, focus on efficient auctions.

$$\max_{p_{ij}, x_{ij}} \underset{\text{2 players}}{2}\left(\beta \underbrace{p_H^e}_{\substack{\text{amount seller gets}\\ \text{if } i \text{ announces } v_H\\ \text{(in expectation)}}} + (1-\beta) p_L^e\right),$$

$p_L^e = \beta p_{LH} + (1-\beta) p_{LL}$

$\underbrace{p_H^e}_{\substack{\text{average amount } i\\ \text{pays if announce } v_H.}} = \beta p_{HH} + (1-\beta) p_{HL}$

s.t. $[\beta x_{HH} + (1-\beta)x_{HL}]v_H - p_H^e \geq 0$ (IR-H)

$[\beta x_{LH} + (1-\beta)x_{LL}]v_L - p_L^e \geq 0$ (IR-L)

$[\beta x_{HH} + (1-\beta)x_{HL}]v_H - p_H^e \geq [\beta x_{LH} + (1-\beta)x_{LL}]v_H - p_L^e$ (IC(HL))

$[\beta x_{LH} + (1-\beta)x_{LL}]v_L - p_L^e \geq [\beta x_{HH} + (1-\beta)x_{HL}]v_L - p_H^e$ (IC(LH))

Problem is to ensure (IR-L) and (IC(HL)) are satisfied. Can ignore the others.

The problem becomes: (if we assume we want to induce the efficient allocation.)

$$\max_{p_{ij},x_{ij}} \; 2(\beta p_H^e + (1-\beta) p_L^e)$$

s.t. $(1-\beta)\frac{1}{2} v_L - p_L^e \geq 0$

$$\beta \tfrac{1}{2} v_H + (1-\beta) v_H - p_H^e \geq (1-\beta)\tfrac{1}{2} v_H - p_L^e$$

- Can show these constraints will be binding.

Get revenue: $\beta v_H + (1-\beta) v_L$

- There is nothing to maximize.

First best: $(1-(1-\beta)^2) v_H + (1-\beta)^2 v_L$

Here, there is an information rent for the buyer:

$$(1-\beta)\tfrac{1}{2}[v_H - v_L]$$

Can reduce the informational rent by looking at optimal (but not necessarily efficient) auctions.

Optimal auction

$$\max_{p_{ij},x_{ij}} \; 2(\beta p_H^e + (1-\beta) p_L^e)$$

s.t. (IR_H), (IR_L), $CI(_{HL})$, $CI(_{LH})$

Here, we will still be able to ignore (IR_H) and $CI(_{LH})$, but we do not impose efficiency.

Recall: The informational rent was:

$$(1-\beta)\tfrac{1}{2}[v_H - v_L] = (1-\beta)x_{LL}[v_H - v_L]$$

Will have (IR_L) and $CI(_{HL})$ binding. Replace p_L^e and p_H^e in the maximand, giving us a problem which is linear in x_{ij}. (subject to feasibility)

- will have a corner solution.

Results:

1. x_{HH} has a positive coefficient $\Rightarrow x_{HH} = \frac{1}{2}$
2. Coefficient on x_{HL} is bigger than coefficient on $x_{LH} \Rightarrow x_{HL} = 1$ and $x_{LH} = 0$
3. Coefficient on $x_{LL} > 0$ iff $v_L \geq \beta v_H$

$$\Rightarrow x_{LL} = \begin{cases} \frac{1}{2} & \text{if } v_L \geq \beta v_H \\ 0 & \text{else} \end{cases}$$

- same condition as in the one-buyer case.

Here, we took the optimal contracting perspective. Can also consider the game theoretic perspective.

English auction $\Leftrightarrow$ Vickrey (second-price sealed bid)
- weakly dominant strategy is to bid one's valuation
  - seller gets revenue: $\beta^2 v_H + (1-\beta^2) v_L$
  - this is worse than even the optimal efficient auction: $\beta v_H + (1-\beta) v_L$
- "Have to be an economist to understand the Vickrey auction."

Optimal auction is better, because it ensures that (IC_{HL}) is binding

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First-price sealed bid auction $\Leftrightarrow$ Dutch auction
- How much does the seller make in this auction?

Equilibrium:
- First observation: v_L bids v_L (Bertrand idea)
- Symmetry: v_L gets the good with prob $(1-\beta)\frac{1}{2}$.
  v_H gets the good with prob. $\frac{1}{2}\beta + (1-\beta)$
- mixed strategy for v_H and indifference with respect to bidding $v_L (+\varepsilon)$, which implies payoff $= (1-\beta)(v_H - v_L)$
- Turns out to have same revenue as first price auction. (in the continuum of types case)

"The sky is the limit, and by 'the sky,' we mean the individual rationality constraint."

(*) What happens if w/pr. p, we play first-price sealed bid and w/pr. 1-p, we play second-price sealed bid?

° In the continuum-of-types case, we will have local incentive compatibility.