Continuum of types adverse selection

$\theta \in [\underline{\theta}, \bar{\theta}]$, $f(\theta)$, $F(\theta)$

$$\max_{\underbrace{q(\theta), T(\theta)}_{\text{revelation principle allows us to look at this}}} \int_{\underline{\theta}}^{\bar{\theta}} [T(\theta) - cq(\theta)] f(\theta) d\theta$$

s.t. $\underline{\theta} V(q(\underline{\theta})) - T(\underline{\theta}) \geq 0$

$$\theta V(q(\theta)) - T(\theta) \geq \theta V(q(\hat{\theta})) - T(\hat{\theta}) \quad \forall \theta, \hat{\theta} \quad (IC)$$

- (IC) $\Longleftrightarrow$ (LIC) & (M), where:

$$\theta V'(q(\theta)) \frac{dq}{d\theta} - T'(\theta) = 0 \qquad (LIC)$$

$$\underbrace{\frac{dq}{d\theta} \geq 0}_{\text{implied by Spence-Mirrlees.}} \qquad (M)$$

- Why is q differentiable? q is weakly increasing, so $q(\theta)$ is differentiable almost everywhere.

Given that $\theta = \arg\max_{\hat{\theta}} \{\theta V(q(\hat{\theta})) - T(\hat{\theta})\}$, we have

$W(\theta) \equiv \max_{\hat{\theta}} \{\theta V(q(\hat{\theta})) - T(\hat{\theta})\}$, so by the envelope theorem, $W'(\theta) = V(q(\theta))$

Thus, $W(\theta) = \underbrace{W(\underline{\theta})}_{=0} + \int_{\underline{\theta}}^{\theta} V(q(x)) dx$

$$\Rightarrow T(\theta) = \theta V(q(\theta)) - W(\theta) = \theta V(q(\theta)) - \int_{\underline{\theta}}^{\theta} V(q(x)) dx$$

The problem then becomes:

$$\max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta v(q(\theta)) - cq(\theta) - \int_{\underline{\theta}}^{\theta} v(q(x))dx\right] f(\theta)\, d\theta$$

$$= \max_{q(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} \Big[\underbrace{\theta v(q(\theta)) - cq(\theta)}_{\text{surplus}} - \underbrace{\frac{1-F(\theta)}{f(\theta)} v(q(\theta))}_{\text{informational rents}}\Big] f(\theta)\, d\theta$$

$$\text{s.t.} \quad \frac{dq(\theta)}{d\theta} \geq 0$$

FOCs:

$$(q(\theta)): \quad \theta v'(q(\theta)) = c + \frac{1-F(\theta)}{f(\theta)} v'(q(\theta))$$

$$\Longleftrightarrow \quad \underbrace{\boxed{\theta - \frac{1-F(\theta)}{f(\theta)}}}_{>0, \text{ otherwise } q(\theta)=0} v'(q(\theta)) = c$$

- may want $q(\theta) = 0$ if $\theta f(\theta)$ too low or $1-F(\theta)$ high.

In order for $\frac{dq(\theta)}{d\theta} \geq 0$ to be satisfied, a sufficient condition is that $\frac{f(\theta)}{1-F(\theta)}$ is increasing.

Price of the marginal unit:

$$\underline{\tilde{T}'(q(\theta))} = \theta v'(q(\theta)) = \frac{\theta c}{\theta - \frac{1-F(\theta)}{f(\theta)}}$$

$$T'(\theta) = \tilde{T}'(q(\theta)) \frac{dq(\theta)}{d\theta}$$

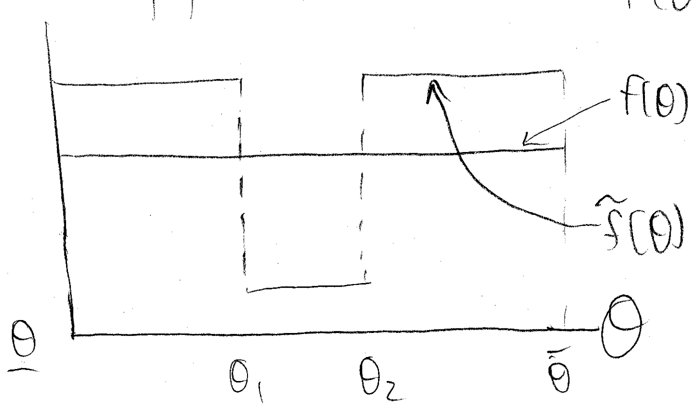
$$\Rightarrow \frac{p-c}{p} = \frac{\dfrac{\theta c}{\theta - \frac{1-F(\theta)}{f(\theta)}} - c}{\dfrac{\theta c}{\theta - \frac{1-F(\theta)}{f(\theta)}}} = 1 - \frac{\theta - \left(\frac{1-F(\theta)}{f(\theta)}\right)}{\theta} = \frac{1-F(\theta)}{\theta f(\theta)}$$

where $p = \tilde{T}'(q(\theta))$

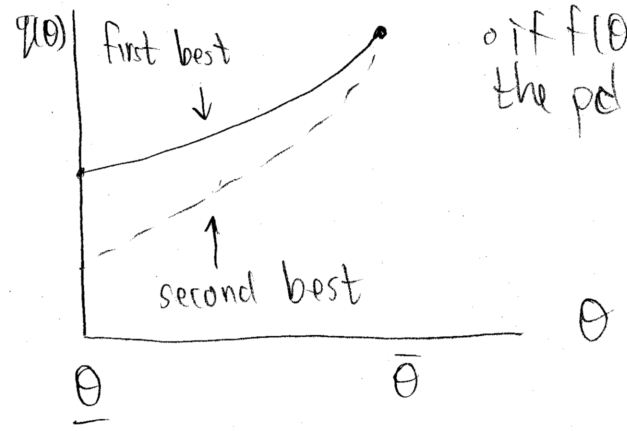
If $\dfrac{f(\theta)}{1-F(\theta)}$ is increasing in θ, $\dfrac{\theta f(\theta)}{1-F(\theta)}$ is increasing in θ, so that $\dfrac{p-c}{p}$ is decreasing in θ.

- quantity discounts (IO implications.)

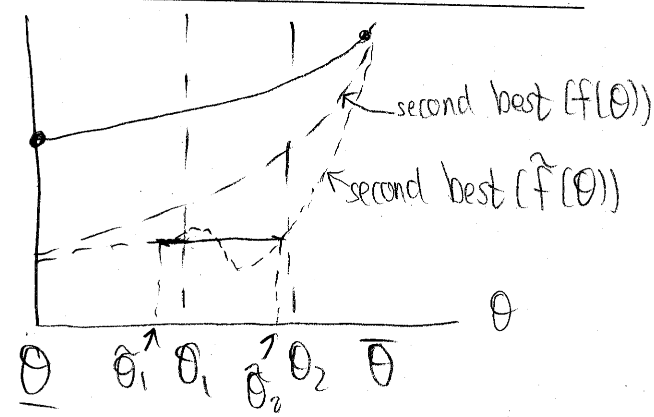
What happens if $\theta - \dfrac{1-F(\theta)}{f(\theta)}$ is not increasing in θ?



- What if the distribution looks like this?
- compute the first best, and the second-best assuming monotonicity is satisfied.



◦ if $f(\theta)$ is the pdf



◦ The second-best for $\hat{f}(\theta)$ will either coincide with the solution ignoring M, (and then $\frac{dq(\theta)}{d\theta} > 0$) or it is flat on intervals.

◦ trading off on the sales to the types on the flat interval with the rents to the types above θ.

◦ This is the bunching and ironing procedure.

◦ If we compute the second-best and find that $\frac{dq(\theta)}{d\theta}$ is decreasing over a range, we will have flats.

Multi-agent adverse selection

◦ efficient auctions
◦ Bargaining with bilateral asymmetric information
◦ optimal provision of public goods.

} mechanism design

auctions with two players and two types per player.

Buyer: $i=1,2$ $v_i \in \{v_L, v_H\}$, $Pr[v_i = v_H] = \beta$
- independent.

Seller: cost $= 0 < v_L$

Single buyer: seller will set price $= v_H$ iff $\beta v_H \geq v_L$ (inefficiency with probability $1-\beta$)

Two buyers, by setting price v_H: inefficiency with probability $(1-\beta)^2$
- is this a good way for the seller to sell?

Optimal <u>efficient</u> auction?

- an auction is defined by $(p_{ij}, \underbrace{X_{ij}}_{\text{pr. [getting the object]}})$, where
  - p_{ij} is the price if the buyers announce v_i, v_j.
  - a game where people choose among message sets S_i and S_j
    - to each (s_i, s_j), there is an associated price and "X" for each buyer.
  - constraints on X_{ij}
    1. $X_{ij} \in [0,1]$ $\forall i,j$
    2. $X_{LL} \leq \frac{1}{2}$, $X_{HH} \leq \frac{1}{2}$, $X_{HL} + X_{LH} \leq 1$

- Efficiency: $1 = 2\bar{x}_{LL} = 2\bar{x}_{HH}$, $\bar{x}_{HL} = 1$, $\bar{x}_{LH} = 0$

Revelation principle: Take any Bayesian game, look at the equilibrium strategies, and call the equilibrium strategies for type $j \in \{L, H\}$ "announcing type j".

Seller will want to:

$$\max 2 [\beta p_L^e + (1-\beta) p_H^e]$$

s.t. (IR) and (IC) constraints.