Outline:

Multi-agent
- RPE in standard model
- Help
- Collusion-proof

Office hours: Monday 6pm

$$q_1 = a_1 + \varepsilon_1 + \alpha\varepsilon_2 \qquad \text{where} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}\right)$$
$$q_2 = a_2 + \varepsilon_2 + \alpha\varepsilon_1$$

Recall: $v = \frac{1}{1+\sigma^2 c \eta}$ in single-agent problem

Contract $w_1(q_1, q_2)$, $w_2(q_1, q_2)$. Principal's problem is:

$$\max E[q_1 + q_2 - w_1(q_1,q_2) - w_2(q_1,q_2) \mid a_1, a_2]$$

- ie principal is risk-neutral.

agents' preferences

- $E[-\exp\{-\eta(w_1(q_1,q_2) - \frac{1}{2}c a_1^2)\} \mid a_1, a_2]$

Consider linear contracts:
- $w_1(q_1, q_2) = z_1 + v_1 q_1 + u_1 q_2$
- $w_2(q_1, q_2) = z_2 + v_2 q_2 + u_2 q_1$

Recall: Certainty equivalent

$E[-\exp\{-\eta[w_1(q_1,q_2) - \frac{1}{2}(a_i^2)]\} \mid a_1, a_2]$

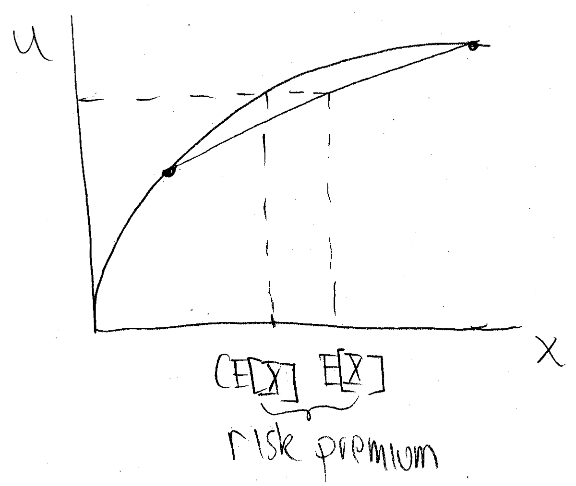
$= -\exp\{-\eta(z_1 + v_1 a_1 + u_1 a_2 - \frac{1}{2}c a_1^2) + \frac{\eta^2}{2}[(v_1+\alpha u_1)^2\sigma_1^2 + (u_1+\alpha v_1)^2\sigma_2^2]\}$

If $\sigma_1^2 = \sigma_2^2 = \sigma^2$, then, this value becomes

$-\exp\{-\eta(z_1 + v_1 a_1 + u_1 a_2 - \frac{1}{2}(a_1^2) + \frac{\eta^2\sigma^2}{2}[(v_1+\alpha u_1)^2 + (u_1+\alpha v_1)^2]\}$

Both players want to maximize the certainty equivalent, taking the other player's action as given:

$$\max_{a_i} \; z_i + v_i a_i + u_i a_{-i} - \frac{1}{2}c a_i^2 - \frac{\eta\sigma^2}{2}[(v_i+\alpha u_i)^2 + (u_i+\alpha v_i)^2]$$



The unique NE is: $a_i^* = \frac{v_i}{c}$

Optimal SB contract solves:

$$\max_{a_i, w_i(\cdot)} E[q_1+q_2-w_1(q_1,q_2)-w_2(q_1,q_2)]$$

s.t. $CE_1 \geq 0$

$CE_2 \geq 0$

$a_1 = \frac{v_1}{c}$

$a_2 = \frac{v_2}{c}$

By symmetry, we want to:

$$\max_{a_1,u_1,v_1,z_1} a_1-[z_1+v_1a_1+u_1a_2]$$

s.t. $0 = z_1+v_1a_1+u_1a_2-\frac{1}{2}ca_1^2-\frac{r\sigma^2}{2}[(v_1+\alpha u_1)^2+(u_1+\alpha v_1)^2]$

$a_1 = \frac{v_1}{c}$

Can write this:

$$\max_{a, w(\cdot)} CE_p+CE_1+CE_2$$

s.t. $CE_1 \geq 0$

$CE_2 \geq 0$

a solves IC constraints.

Two steps:

1) For a given v_1, use u_1 to minimize the variance

[2] Choose optimal v_1

<u>Step 1</u>: (u_1): $2(v_1 + \alpha u_1)\alpha + 2(u_1 + \alpha v_1) = 0$

$$\Rightarrow u_1 = -\frac{2\alpha}{1+\alpha^2} v_1$$

- If $\alpha > 0 \Rightarrow$ penalize if other agent produces more, since own output is more likely to have resulted from positive common shock.

<u>Step 2</u>: Taking FOCs wrt v_1 after substituting the result from step 1 gives

$$v_1 = \frac{1+\alpha^2}{1+\alpha^2 + \eta c \sigma^2 (1-\alpha^2)^2}$$

- use more information to reduce variance
- If $\alpha = 1$, this is a common shock that we can filter out, giving us:
  - $v_1 = 1$
  - $u_1 = -1$

<u>Suppose agent 1 can "help" agent 2</u>:

- $q_1 = a_1^A + \varepsilon_1$
- $q_2 = a_2^B + \varphi a_2^A + \varepsilon_2$
- $C(a_1^A, a_2^A)$ - cost of effort for agent A is convex

assume $\varepsilon_1, \varepsilon_2$ are iid

- No common shocks $\Rightarrow u_B = 0$
- $W_B(q_1, q_2) = z_B + v_B q_2 + u_B q_1$
- Do we want $u_A \neq 0$?
  - Depends on C_{12}, φ, and risk premium.

Suppose $C(a_1, a_2) = c_1 \frac{a_1^2}{2} + c_2 \frac{a_2^2}{2} + c_{12} a_1 a_2$

alternatively, think about $C(a_1^A, a_2^A) = (a_1^A + a_2^A)^2$ and $\varphi = 1$. Then must have $u_A = v_A$.

- Must have $a_1^A + a_2^A = v_A = u_A$

The risk premium must be paid for each task that incentives are provided for, regardless of the effort level chosen.

- Don't necessarily want to induce effort on both tasks.

Joll side-contracting

3 cases:

$T^1 = T(q_1, q_2)$

$T^2 = T(a_1, a_2)$

$T^3 = T(a_1, a_2, q_1, q_2)$

$$\max \; s_1 a_1 + s_2 a_2 - c(a_1) - \frac{\eta}{2}\sigma^2[s_1^2 + s_2^2]$$

$$\text{s.t. } (1-s_1)a_1 + (1-s_2)a_2 - c(a_2) - \frac{\eta}{2}\sigma^2[(1-s_1)^2 + (1-s_2)^2] \geq \text{something}$$

since $w_1 + w_2 = v_1 q_1 + u_1 q_2 + v_2 q_2 + u_2 q_1 + z_1 + z_2$
$= (v_1 + u_2) q_1 + (v_2 + u_1) q_2 + z_1 + z_2$

Thus, we need $v_1 + u_2 = v_2 + u_1 = 1$.

We want to minimize the sum of the two risk premiums, in some sense. (We actually want to minimize the harmonic mean of the risk premiums:
$\underbrace{HM(x,y)}_{\text{harmonic mean}} = \frac{1}{x} + \frac{1}{y}$)

- It is like contracting with one agent.