Multi-Tasking

2 dimensions $q_1 = a_1 + \varepsilon_1$ $\varepsilon_1 \sim N(0, \sigma_1^2)$
$q_2 = a_2 + \varepsilon_2$ $\varepsilon_2 \sim N(0, \sigma_2^2)$

Risk-neutral principal

- $E[q_1 + q_2 - w(q_1, q_2) \mid a_1, a_2]$

CARA agent with monetary cost:

- $C(a_1, a_2) = \frac{c_1}{2} a_1^2 + \frac{c_2}{2} a_2^2 + c_{12} a_1 a_2$
- $C_{12}(a_1, a_2) = c_{12} \in [\underbrace{0}_{a_1, a_2 \text{ perfect complements}}, \underbrace{\sqrt{c_1 \cdot c_2}}_{a_1, a_2 \text{ perfect substitutes}}]$
- $\bar{u} = -\exp\{-\eta(0)\} = -1$

Linear contracts

- $w(q_1, q_2) = t + s_1 q_1 + s_2 q_2$

$E[-\exp(-\eta X)]$, $X \sim N(\mu, \sigma^2)$

$= -\exp\{-\eta(\mu + \eta \frac{\sigma^2}{2})\} \geq -\exp\{-\eta(0)\}$

$\Leftrightarrow$ $\mu + \eta \frac{\sigma^2}{2} \geq 0$

$E[-\exp\{-\eta(w(q_1, q_2) - c(a_1, a_2))\}] \geq \bar{u}$

$$\Rightarrow \underbrace{t + s_1 a + s_2 a_2}_{\text{expected wage}} - \underbrace{\eta[s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2]}_{\text{risk premium}}$$

$$- \underbrace{\left[\frac{c_1}{2} a_1^2 + \frac{c_2}{2} a_2^2 + c_{12} a_1 a_2\right]}_{\text{cost of effort}} \geq \bar{u} \qquad (IR)$$

Want to:

$$\max \ (1-s_1) a_1 + (1-s_2) a_2 - t$$

$$\text{s.t. } t + s_1 a_1 + s_2 a_2 - \frac{\eta}{2}[s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2]$$

$$- \left[\frac{c_1}{2} a_1^2 + \frac{c_2}{2} a_2^2 + c_{12} a_1 a_2\right] \geq 0 \quad (IR)$$

$$(a_1): \ s_1 - c_1 a_1 - c_{12} a_2 = 0 \qquad (IC\text{-}a_1)$$

$$(a_2): \ s_2 - c_2 a_2 - c_{12} a_1 = 0 \qquad (IC\text{-}a_2)$$

$$\Rightarrow a_2 = \frac{s_2 - c_{12} a_1}{c_2}$$

$$\Rightarrow s_1 - c_1 a_1 - c_{12} \frac{s_2 - c_{12} a_1}{c_2} = 0$$

$$\Rightarrow a_1 = \frac{s_1 c_2 - s_2 c_{12}}{c_1 c_2 - c_{12}^2}$$

$$a_2 = \frac{s_2 c_1 - s_1 c_{12}}{c_1 c_2 - c_{12}^2}$$

$$\max_{s_1, s_2} \Omega(s_1, s_2 | \alpha). \text{ Let } (s_1^*(\alpha), s_2^*(\alpha)) \in \arg\max_{s_1, s_2} \Omega(s_1, s_2 | \alpha).$$

<u>Thm</u>: If
- $\dfrac{\partial^2 \Omega}{\partial s_1 \partial s_2} \geq 0$
- $\dfrac{\partial^2 \Omega}{\partial s_2 \partial \alpha_i} \geq 0$, then
- $\dfrac{\partial^2 \Omega}{\partial s_1 \partial \alpha_i} \geq 0$

$$\frac{\partial s_1^*}{\partial \alpha_i} \geq 0, \quad \frac{\partial s_2^*}{\partial \alpha_i} \geq 0$$

Substituting in the IR constraint into the objective function:

$$\max_{s_1, s_2} a_1 + a_2 - \frac{\eta}{2}\left[s_1^2 \sigma_1^2 + s_2^2 \sigma_2^2\right] - \left[\frac{c_1}{2} a_1^2 + \frac{c_2}{2} a_2^2 + c_{12} a_1 a_2\right]$$

$$\text{s.t. } a_1 = \frac{s_1 c_2 - s_2 c_{12}}{c_1 c_2 - c_{12}^2}$$

$$a_2 = \frac{s_2 c_2 - s_1 c_{12}}{c_1 c_2 - c_{12}^2}$$

- $\text{sign}\left(\dfrac{\partial^2 \Omega}{\partial s_1 \partial s_2}\right) = \text{sign}(c_{12}) \geq 0$
- $\text{sign}\left(\dfrac{\partial^2 \Omega}{\partial s_1 \partial (-\sigma_1^2)}\right) = \text{sign}(\eta s_1 \sigma_1^2) \geq 0$
- $\text{sign}\left(\dfrac{\partial^2 \Omega}{\partial s_2 \partial (-\sigma_1^2)}\right) = \text{sign}(0) = 0$

Result:

- If $\sigma_1^2 \uparrow \Rightarrow s_1 \downarrow$
- If $\sigma_1^2 \uparrow \Rightarrow s_2 \downarrow$ by supermodularity

$$\frac{\partial s_1^*}{\partial(-\sigma_1^2)} \geq 0 \iff \frac{\partial s_1^*}{\partial \sigma_1^2} \leq 0$$

$$\frac{\partial s_2^*}{\partial(-\sigma_1^2)} \geq 0 \iff \frac{\partial s_2^*}{\partial \sigma_1^2} \leq 0$$

Task Exclusion

- Principal cares about q_1
- agent cares about q_2
- might want to exclude a certain task

Job Design

- $q_i = a_i + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma_i^2)$

  and $c(a_1, a_2) = (a_1 + a_2)^2$
- want to group tasks that are "measurable" (i.e. σ_i^2 small)

Can show that there is a cut-off σ_2^* s.t.

$\sigma_i^2 < \sigma_2^* \Rightarrow$ put both tasks together

$\sigma_i^2 > \sigma_2^* \Rightarrow$ put this task with other agent.

Advocates and Conflicts

Suppose $q_1 = a_1 - \eta a_2 + \varepsilon_1$
$q_2 = a_2 - \eta a_1 + \varepsilon_2$

- Do we want these tasks to be grouped?

Baker: No uncertainty. Misalignment of incentives

$V(a_1, a_2) = v_1 a_1 + v_2 a_2$

- can only observe $P(a_1, a_2) = p_1 a_1 + p_2 a_2$
- can only contract on P:
  - e.g. $t + sP$. How do we find s?

Can show that

$$s = \frac{v_1 f_1 + v_2 f_2}{f_1^2 + f_2^2} = \frac{\|v\| \times \|f\| \cos(\gamma)}{\|f\|^2}$$

Next problem set due in a week.