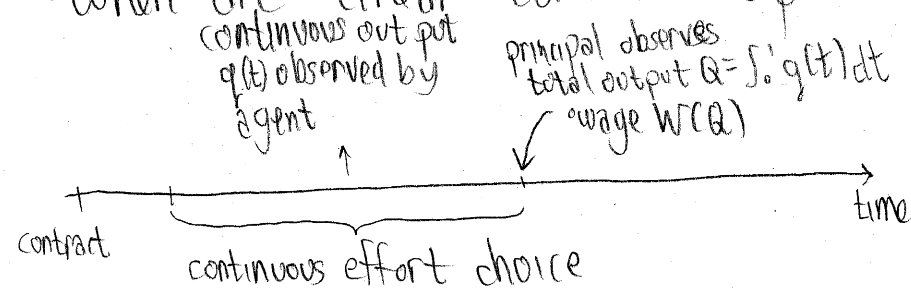
⚠ Error last time:

$$\text{Loss} \leq \frac{1}{M}\int_{-\infty}^{\underline{q}} [u(K) - u(w^*)] f_a(q|a^*)\,dq$$

$$= \frac{1}{M}\Big[\underbrace{-\int_{-\infty}^{+\infty} u(w^*) f_a(q|a^*)\,dq + \Psi(a^*)}_{\text{finite}}\Big] \to 0 \text{ as } M \to +\infty$$

Next week:
- Johannes on Thursday
- Mathias on Friday
- No class on Tuesday ☺

Holmstrom - Milgrom (EMA '87)
- When are linear contracts optimal?



- at each moment in time, the agent must choose an effort intensity level.

- assume instantaneous effort cost $\Psi(e_t)$ is convex
- want to induce constant level of effort in this interval.

With constant absolute risk aversion, linear incentives are optimal.

<u>Multi-agent moral hazard</u> (Holmstrom - Moral Hazard in Teams)

Two ways of looking at this:

1. Only aggregate output is observable
2. Individual outputs are observable

In the [1] case, having multiple agents hurts. In the [2] case, having multiple agents helps. (Relative performance evaluation)

Case [1]:

- $q = q[a_1, a_2, \ldots, a_n]$
- First best: choose a^* s.t. $\frac{\partial q(a^*)}{\partial a_i} = \psi_i'(a_i^*)$ $\forall i$
- Can a "partnership" deliver this?
  - $\sum_{j=1}^{n} w_j(q) = q$ $\forall q$ $\Rightarrow$ $\sum_{j=1}^{n} \frac{dw_j(q)}{dq} = 1$
- For first best, need $\frac{\partial w_i(q)}{dq}\frac{\partial q(a)}{\partial a_i} = \psi_i'(a_i)$

  with $\frac{dw_i(q)}{dq} = 1$ $\forall i$.
- Partnership provides underprovision of effort
  - one idea: bring in a "budget breaker"
  - to have 1st best, need $\frac{dw_{BB}(q)}{dq} = -(n-1)$
  - Then, can have $\frac{dw_j(q)}{dq} = 1$ $\forall j = 1, \ldots, n$ and

$$\sum_{j=1}^{n} \frac{dw_j(q)}{dq} + \frac{dw_{BB}(q)}{dq} = n - (n-1) = 1$$

Case 2] with normal noise and CARA utility will be covered in recitation.

- $q_1 = a_1 + \varepsilon_1 + \beta \varepsilon_2$ ◦ $E[\varepsilon_1 \varepsilon_2] = 0$
- $q_2 = a_2 + \varepsilon_2 + \beta \varepsilon_1$ ◦ β is common knowledge
  - when $\beta > 0$, want to reward individual i not only for a high q_i, but also for a low q_j. (relative performance evaluation)
    - Want to give you more credit when you do "better" than the other person. This helps filter out the common shock.
    - This is consistent with the Holmstrom result on the use of additional information.

Tournaments

- Get a fixed wage z plus a "prize" w if your output is the highest.
  - e.g. internal promotion
- Lazear-Rosen: risk neutrality and no common shocks
  - $q_i = a_i + \varepsilon_i$ $i \in \{1,2\}$, $\psi(\cdot)$ convex cost of effort
  - $\varepsilon_i \sim F(0, \sigma^2)$ independent across i
- First-best: $\psi'(a_i) = 1$
  - one way to reach the first best is

a piece rate: $w_i = z_i + q_i$

- another way is a tournament: each agent obtains $z_i + P \underbrace{W}_{\text{prize}} - \psi(a_i)$, where $P = Pr[q_i > q_j]$

$$\Rightarrow P = Pr[a_i - a_j > \varepsilon_j - \varepsilon_i] \equiv \underbrace{H(a_i - a_j)}_{\text{cdf of } \varepsilon_j - \varepsilon_i}$$

- $\varepsilon_j - \varepsilon_i$ has mean 0 and variance $2\sigma^2$.
- it is riskier, but we have risk neutrality.

- agent: $\max_{a_i} z_i + H(a_i - a_j) W - \psi(a_i)$

$$(a_i): \quad \frac{dH(a_i^* - a_j)}{\partial a_i} \cdot W = \psi_i'(a_i^*)$$

$$\Leftrightarrow \quad h(a_i^* - a_j) \cdot W = \psi_i'(a_i^*)$$

- assuming symmetry ($\psi_i = \psi$) this FOC becomes:

$$\psi'(a_i^*) = h(0) \cdot W$$

- Recall: FB: $\psi'(a_i^*) = 1$

$\Rightarrow$ If we let $W = \frac{1}{h(0)}$, we can achieve FB

For next Friday, have a look at Green-Stokey

- take Lazear-Rosen, add risk aversion and common shock. When is relative performance better? When are tournaments better?
- Tournaments with many agents can do very well.
  - can approximate second-best.