Outline:
(*) Evaluation
(*) Incentives vs insurance
(*) Value of information

Incentives vs. Insurance

- agent exerts effort a
- output q with density $f(q,a)$
  - or $q = a + \theta$, $\theta \sim N(0, \sigma^2)$

Principal. First best problem

$$\max_{a,\, w(\cdot)} \int v(q - w(q)) f(q,a)\, dq$$

$$\text{s.t.} \int u(w(q)) f(q,a)\, dq - \psi(a) \geq \underline{u} \quad (IR, \lambda)$$

- want to provide incentives and insurance
  - have two instruments: a, $w(\cdot)$

assuming the first order approach is valid, we have the following:

$$\int u(w(q)) f_a(q,a)\, dq - \psi'(a) \quad (IC, \mu)$$

- Now, we have two objectives and one instrument.

First order conditions of this problem:

$$(w(q)):\ \frac{v'(q-w(q))}{u'(w(q))} = \lambda + \mu \frac{f_a(q,a)}{f(q,a)} \quad \forall q$$

if principal is RN $\Rightarrow$

$$\frac{1}{u'(w(q))} = \lambda + \mu \frac{f_a(q,a)}{f(q,a)} \quad \forall q$$

If there are only two possible actions: $a \in \{a_L, a_H\}$

$$\frac{1}{u'(w(q))} = \lambda + \mu \frac{f(q,a_H) - f(q,a_L)}{f(q,a_L)}$$

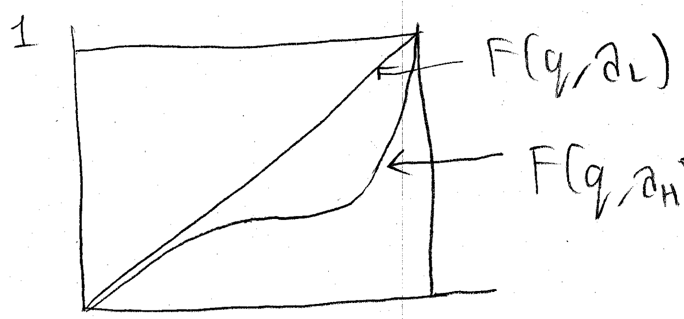
MLRP: The higher q, the more likely it is it is "caused" by a higher level of effort.

MLRP implies a monotone incentive scheme.

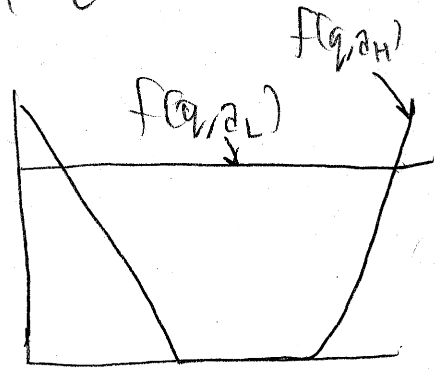
Why induce effort? FOSD: $F_a \leq 0$

Why is wage schedule in q? MLRP

- High q is a signal for high a.



$\Leftrightarrow$



- Here, FOSD holds, but MLRP does not

Signal $y \longrightarrow f(q, y, a)$

- can contract on q and y

Example: $y = q_{BOB} = a_{BOB} + \theta_{BOB}$

$\text{cov}(\theta_{ANN}, \theta_{BOB})$

$q = a_{ANN} + \theta_{ANN}$

First order condition becomes:

$$\frac{1}{u'(w(q,y))} = \lambda + \mu \frac{f_a(q,y,a)}{f(q,y,a)}$$

$w(q,y)$ depends on y iff $\dfrac{f_a(q,y,a)}{f(q,y,a)}$ depends on y.

What is the value of having a signal? When can we rank these signals?

Suppose θ is the state of nature (unknown)

a - action

$x(a,\theta) \Rightarrow \tilde{u}(x(a,\theta)) \equiv u(a,\theta)$

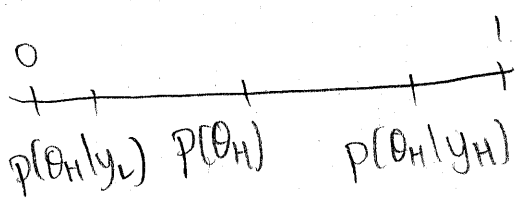
y - signal. $(y,\theta) \sim p(y,\theta)$

1. $p(\theta)$ prior
2. $p(y|\theta)$ likelihood
3. $p(\theta|y)$ posterior

- $$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{\int p(y|\tilde{\theta})p(\tilde{\theta})d\tilde{\theta}}$$

- $E_y[p(\theta|y)] = p(\theta)$

Suppose we have $\theta_H, \theta_L, y_H, y_L$



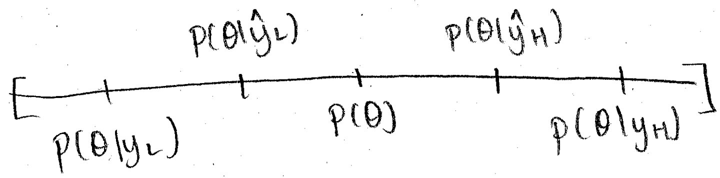
- $p(\theta_H|y_H)p(y_H) + p(\theta_H|y_L)p(y_L) = p(\theta_H)$

Idea: signal is informative ⇒ moves the posteriors
signal is valuable ⇒ changes the decision

1. Sufficient statistic: $T(y)$ is a sufficient statistic for θ wrt y iff
$$p(y|\theta) = p(y|T(y))\,p(T(y)|\theta)$$
- $$p(\theta|y) = \frac{\cancel{p(y|T(y))}\,p(T(y)|\theta)\,p(\theta)}{\int \cancel{p(y|T(y))}\,p(T(y)|\tilde{\theta})\,p(\tilde{\theta})d\tilde{\theta}}$$

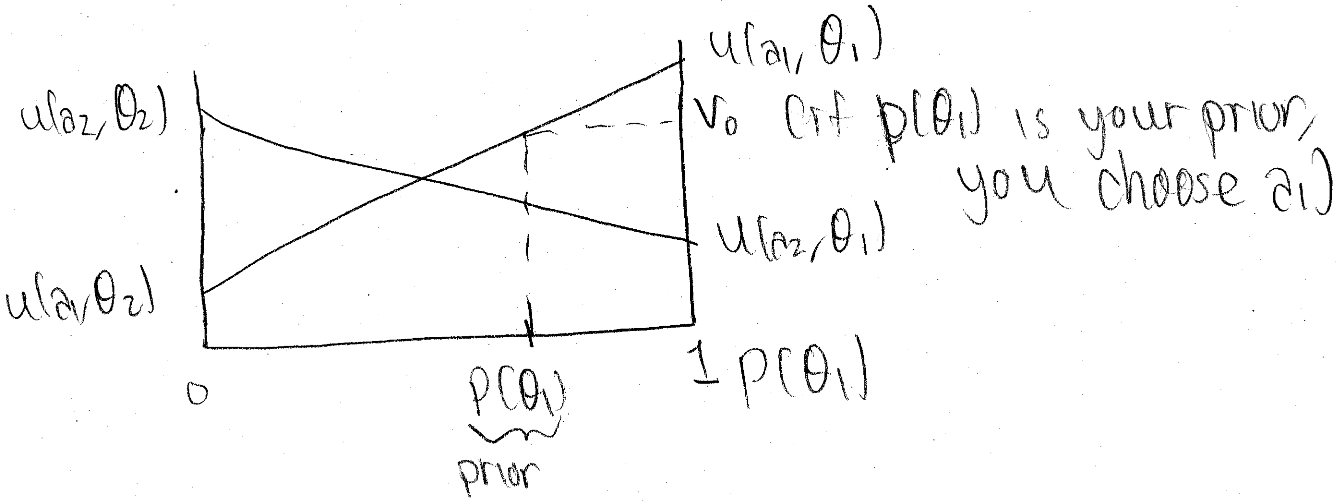
$$= \frac{p(T(y)|\theta)\,p(\theta)}{\int p(T(y)|\tilde{\theta})\,p(\tilde{\theta})\,d\tilde{\theta}}$$

Blackwell Theorem: a signal y is more informative than a signal $\hat{y}$ if the posteriors of y are mean preserving spread of the posteriors of $\hat{y}$ iff $\hat{y}$ is a "garbling" of y



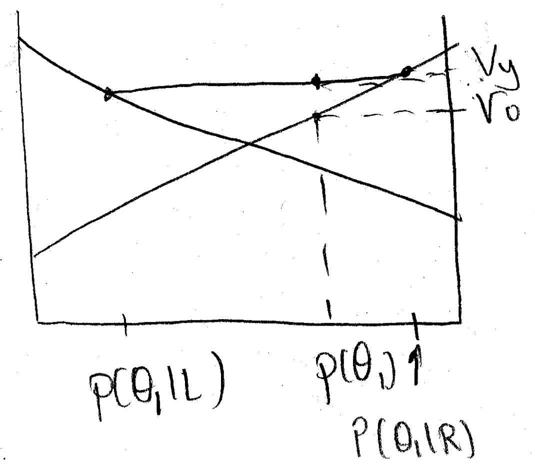
Uninformed: Choose $a \in \{a_1, a_2\}$ to maximize $E_\theta[u(a|\theta)]$ with $\theta \to \theta_1$, $\theta \to \theta_2$

$$V_0 = \max_a \; p(\theta_1)u(a,\theta_1) + p(\theta_2)u(a,\theta_2)$$



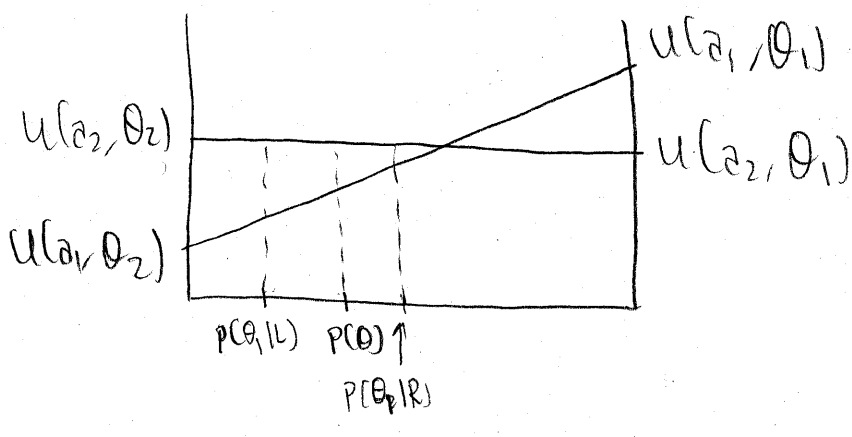
Signal $y \in \{L, R\}$. Condition your action on the signal: $a(L), a(R)$

$$\max_{a(\cdot)} \left[p(\theta_1|R)u(a(R),\theta_1) + p(\theta_2|R)u(a(R),\theta_2) \right] p(R) + \left[p(\theta_1|L)u(a(L),\theta_1) + p(\theta_2|L)u(a(L),\theta_2) \right] p(L)$$



Value of information: $V_y - V_0$

y is more valuable than $\hat{y}$ if
$V_y - V_0 > V_{\hat{y}} - V_0$



- here, $y \in \{R, L\}$ is not valuable, because it does not change your action.