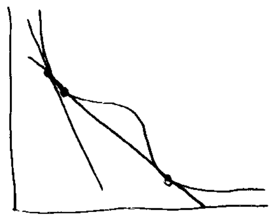
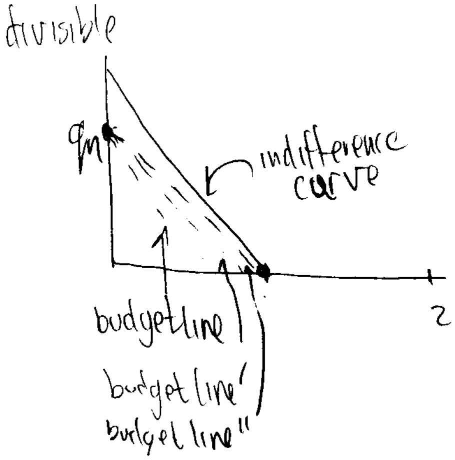


Demand correspondence is upper hemi-continuous

$D(p) \subseteq \mathbb{R}^2$

Suppose $p_n \to p$ and $q_n \in D(p_n)$ with $q_n \to q$. Then $q \in D(p)$.



- indivisible commodity and divisible commodity
  - Demand correspondence is <u>not</u> upper hemicontinuous

This was first noted by Mas-Colell 1974

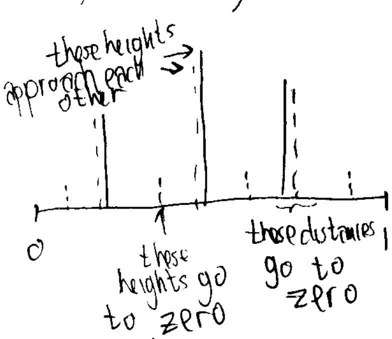
$$\mu^n = \sum_{k \in [0,1]} \alpha_k^n \delta_k \quad , \alpha_k \in \{0,1,2,\ldots,K\}$$

- commodity bundle (differentiated commodities)

$u(\mu)$ was defined wrt weak* topology

$\mu_n \to \mu \iff \forall f : [0,1] \to \mathbb{R}$ continuous

$$\langle \mu_n, f \rangle \to \langle \mu, f \rangle$$
$$\int f \, d\mu_n \to \int f \, d\mu$$
$$\sum f(k) \alpha_k^n \qquad \sum f(k) \alpha_k$$



Weak* continuity of preferences:

$u(\mu_n) \to u(\mu)$

This problem is not so serious with respect to teams ie membership is indivisible, but the "goal" is the associated income, not the utility of membership.

The weak* topology is the topology associated with the idea of substitutes.

- The commodity space here is approximately finite-dimensional.

What if we put incentive constraints into the team production model?

Incentives

(z, T, s) relevant for consumer $v(z, T, s)$

(y, T, s) relevant for producer, $y \in Y(T, s)$

Teams only model:

$v_i(T, s)$ $\quad (T, s) \in B_i$ $\quad \bigcup_i B_i = B$

$x_i(B_i)$ $\quad x_0(B)$

$T \to v_i(T, s)$ game in normal form

$\searrow S(T) \to v_i : S(T) \to \mathbb{R}$

Lotteries: $A_i = Z_i \times B_i$. we moved to $L(A_i)$, the lottery interpretation.

Let $L(T)$ be the set of mixed strategies.

$$L(T) = \{ \mu : \mu_s \geq 0, \sum_{s \in S(T)} \mu_s = 1 \}$$

$$V_i^*(P_i) = \max \{ \Sigma v_i(T, s) - P_i(T, s) : (T, s) \in B_i \}$$

$$= \max \{ \langle v_i, \mu \rangle - \langle p_i, \mu \rangle : \mu \in \bigcup_{T \in \mathcal{T}_i} L(T) \}$$

Without incentive constraints, under quasilinearity, lotteries do not do anything.

$S(T) = S_1(T) \times \dots \times S_T(T)$

Let $d_i : S_i(T) \to S_i(T)$ be the deviation function

$d_i(s_i) = s_i'$, s_i' may be $\neq s_i$

$$\{\mu : \sum_s [v_i(d_i(s_i), s_{-i}) - v_i(s_i, s_{-i})] \mu(s_i, s_{-i}) \leq 0\} \equiv C_i(T)$$

L This gives the expected gain from following the deviation strategy d_i.

$C_i(T)$ is the set of mixtures incentive compatible for i. Define $C(T) = \bigcap_{i \in T} C_i(T)$. This is the set of correlated equilibria for the game. Marco will show us that this is nonempty. (It is related to the existence of feasible solutions to a linear programming problem.)

What if we restrict ourselves in such a way that the feasible set of actions is $C(T)$.

Let $C = \bigcup_T C(T)$ $\qquad x = (x_0, x_1, \dots, x_n) \in M[C] \times M[L_1] \times \dots \times M[L_n]$

$L_i(T) = \bigcup_{T \in \mathcal{T}_i} L(T)$

If x is feasible,

$$\sum_{\mu \in L_i} x_i(\mu) = r_i$$

$$x_0(\mu) - x_i(\mu) = 0 \quad \forall \mu \in C_i$$

- Individuals are choosing over L_i
- The producer can only offer contracts over C_i
- price-taking will guide us toward incentive compatible allocations. (entrepreneurs will only offer $\mu \in C_i$)

$$\max\{\sum \langle v_i, x_i\rangle : \sum_{\mu \in L_i} x_i(\mu) = r_i,\ x_0(\mu) - x_i(\mu) = 0\ \forall \mu \in C_i\}$$

$$\leq \max\{\sum \langle v_i, x_i\rangle : \sum_{\mu \in L_i} x_i(\mu) = r_i,\ x_0(\mu) - x_i(\mu) = 0\ \forall \mu \in L_i\}$$

└ first best vs. second best.

Formally, though, we have not changed the structure of the problem at all.

<u>Without incentive constraints:</u>

$$\sum_{i \in T} P_i(T,s) \geq 0 \quad \text{(no arbitrage duality condition)}$$

$$\Rightarrow \langle \sum_{i \in T} P_i, \mu\rangle \geq 0 \quad \forall \mu \in L(T)$$

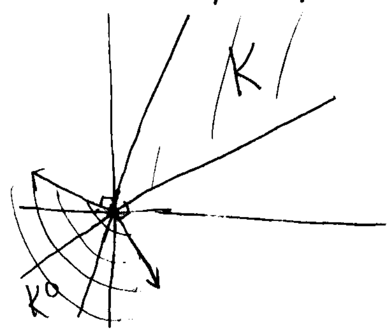
<u>With incentive constraints:</u>

$$\langle \sum P_i, \mu \rangle \geq 0 \quad \forall \mu \in C(T)$$

- smaller set of feasible mixtures

Suppose K is a convex cone. Let K^{0} be the polar of K:

$$K^{0} = \{q : qK \leq 0\} = \{q : qk \leq 0 \ \forall k \in K\}$$



The smaller K is, the larger K^{0} is.

The more restrictive the incentive compatibility constraint, the more possible pricing schemes.

- can make better use of punishments.

$$x_0(\mu) > 0 \Rightarrow \langle \Sigma P_i, \mu \rangle = 0 = \max\{\langle \Sigma P_i, \mu' \rangle : \mu' \in V(C(T))\}$$

$$x_i(\mu) > 0 \Rightarrow \langle v_i, \mu \rangle - \langle P_i, \mu \rangle = v_i^*(P_i)$$

Incentive constraints do not necessarily lead to mixtures.