$x_i \in M[Z_i \times B_i]$

2 interpretations:

- $x_i(z,b)$ - mass of people at (z,b)
- $x_i(z,b) \longrightarrow \lambda^i(z,b) = \frac{x_i(z,b)}{\sum_{(z',b')} x_i(z',b')}$ lottery

For the quasilinear model, it does not matter which interpretation you use, because associated with $x_i(z,b)$ are money payments $m_i = -pz + P_i(b)$

$\max_{(z',b')} \{ v_i(z',b') - pz' + P_i(b') \}$ - you are restricted to no lotteries (1)

$\max_{(z',b')} \{ \underbrace{\langle v_i, \lambda \rangle}_{= \sum v_i(z,b)\lambda(z,b)} - \underbrace{\langle (p, P_i), \lambda \rangle}_{\text{weighted average of money payments}} \}$ (2)

Can let $\lambda = \delta_{(b,z)}$; (1) is special case of (2)

It turns out there are no gains to lotteries.

Let $\lambda = \underset{(z',b')}{\operatorname{argmax}} \{ v_i(z',b') - pz' + P_i(b') \}$

Suppose we have NTU

- ie we insist $\langle (p, P_i), \lambda \rangle = 0$

First look at TU economies:

$v_i(q_i)$ concave

$$\max\left\{\sum_{i=1}^{n} v_i(q_i) : \sum_{i=1}^{n} q_i = 0\right\} = \sum_{i=1}^{n} v_i(\bar{q}_i)$$

By concavity $\exists p \in \partial v_i(\bar{q}_i) \quad \forall i$

$\Rightarrow v_i^*(p) = v_i(\bar{q}_i) - p\bar{q}_i$

$v_j^*(p) = v_j(\bar{q}_j) - p\bar{q}_j$

What if $p\bar{q}_i = 0$?

$D_i(p; v_i) = \operatorname{argmax}\{v_i(q) : pq = 0\}$

demand for i given utility is v_i

If we can find money transfers s.t. $m_i - pq_i = 0 \ \forall i$, then we are in the NTU economy.

This is sensitive to transformations because of the concavity assumption

Suppose $p\bar{q}_i > 0$, $p\bar{q}_j < 0$. Then v_i

(I) $\sum_{i=1}^{n} v_i(\bar{q}_i) = \max\left\{\sum_i v_i(q)x_i(q) : \sum_q x_i(q) = 1 \ \forall i \ \sum_q qx_i(q) = 0\right\}$

(i,q) activity

Imposing the restriction $x_i \in \{0,1\}$ is not binding if v_i is concave

$(i,q) \to v_i(q)$ payoff

$x_i(q)$ level of operation

(II) $\min\left\{\sum 1\cdot\pi_i : \pi_i + pq \geq v_i(q) \ \forall (i,q)\right\} = \min_p \left\{\sum_{i=1}^{n} v_i^*(p)\right\}$

$\Leftrightarrow \pi_i \geq v_i(q) - pq$

$\pi_i = v_i^*(p)$

In general, $pq_i \neq 0 \quad \forall i$

The person whose $pq_i > 0$ has a better ability to "produce utility from q_i"

Now, $\max_\lambda \{ \sum_{i=1}^n \lambda_i v_i (q_i) : \sum_{i=1}^n q_i - \omega \} = \sum_{i=1}^n \lambda_i v_i (q_i(\lambda))$

We will get some $p(\lambda)$

Looking for a $\lambda \gg 0$ s.t. $p(\lambda) \cdot q_i(\lambda) = 0 \quad \forall i$

Let $\Lambda = \{ (\lambda_i) : \lambda_i \geq 0, \sum_{i=1}^n \lambda_i = 1 \}$

Does $\exists \lambda \in \Lambda$ with $\lambda \gg 0$ s.t. $p(\lambda) q_i(\lambda) = 0 \quad \forall i$?

Yes, by a fixed point theorem

$h(c) = \max \{ cx : Ax \leq b, x \geq 0 \} \Rightarrow$ we can find the set of all solutions for all λ.

Define $\operatorname{dom} v_i = \{ q : v_i(q) > -\infty \} \equiv S$

$$I_S(q) = \begin{cases} 0 & \text{if } q \in S \\ -\infty & \text{if } q \notin S \end{cases}$$

Let $\Phi_i(\lambda) = \dfrac{\lambda_i + \max\{p(\lambda) q_i(\lambda), 0\}}{1 + \sum_j \max\{p(\lambda) q_j(\lambda), 0\}}$, $\Phi = [\Phi_i(\lambda)]$

$\Phi : \Lambda \rightrightarrows \Lambda$. We will want to apply Kakutani's fixed point theorem.