$V_i : Z_i \times B_i \to \mathbb{R}$, Z_i convex,

$B_i = \{(T, s) : \ldots\}$ finite set

$b \in B_i$ associate with b a measure $\delta_b \in M[B_i]$

where $\delta_b(E) = \begin{cases} 1 & b \in E \\ 0 & b \notin E \end{cases}$ where E is measurable

$\text{Prob}[B_i] = \{ \beta = \sum_{b \in B_i} \gamma_b \delta_b : \gamma_b \geq 0, \sum \gamma_b \delta_b = \beta, \sum \gamma_b = 1\}$

Let $f : \mathbb{R}^n \to \mathbb{R} \cup \{-\infty\}$ Define $\hat{f}(z) = \sup\{ \sum_k \lambda_k f(z_k) : \lambda_k \geq 0, \sum_k \lambda_k = 1, \sum_k \lambda_k z_k = z\}$

f need not have a convex domain, but by definition, $\hat{f}$ does have a convex domain.

$\hat{V}_i : Z_i \times \text{Prob}[M_i] \to \mathbb{R}$

$\hat{V}_i(\underset{Z_i}{\underset{\cap}{w}}, \underset{\text{Prob}[M_i]}{\underset{\cap}{\beta}}) = \max\{ \sum_{(z,b)} \gamma_{(z,b)} V_i(z,b) : \gamma_{(z,b)} \geq 0, \sum_{(z,b)} \gamma_{(z,b)} = 1, \sum_{(z,b)} \gamma_{(z,b)} (z, \delta_b) = (w, \beta)\}$

Two interpretations

1] Lottery interpretation

2] Utility function for the <u>type</u> of individual

- $\hat{V}_i(w, \beta) = \sum_{(z,b)} \underbrace{\gamma_{(z,b)}}_{\text{fraction of individuals consuming } (z,b)} V_i(z,b)$, where $\sum_{(z,b)} \gamma_{(z,b)} (z, \delta_b) = (w, \beta)$

Lottery interpretation

$\underset{\text{expected utility}}{E_\gamma V} \equiv \langle V_i, \gamma \rangle = \sum_{(z,b)} V_i(z,b) \gamma_{(z,b)}$

An allocation $x = (x_0, x_1, \ldots, x_n)$ is feasible, where $x_i \in M[Z_i \times B_i]$,

$\gamma^i(z,b) = \dfrac{x_i(z,b)}{\sum_{(z',b')} x_i(z',b')}$

- Everyone is choosing the same lottery in this interpretation

$r_i \, \gamma^i(z,b) = x_i(z,b)$

- Each individual need not be consuming the same bundle.

- Each individual of type i must be receiving the same utility, since they are all facing the same prices.

$v_i(z^0,b^0) - pz^0 + P_i(b^0) = v_i(z^1,b^1) - pz^1 + P_i(b^1)$

What if, instead of assigning $\frac{1}{2}$ of the population to (z^0,b^0) and $\frac{1}{2}$ to (z^1,b^1), we give each person the lottery $(\frac{1}{2}z^0 + \frac{1}{2}z^1, \frac{1}{2}\delta_{b^0} + \frac{1}{2}\delta_{b^1})$?

- They will get the same utility with quasilinearity
  - There is effectively no risk aversion with quasilinearity
  - The standard model departs from this. Here, the interpretation definitely matters.

Moving away from the quasilinear model: 2 ways: $\boxed{1}$ Change $U_i(z,b,m) = v_i(z,b) + m$ to $v_i(z,b)$

$v_i(z,b)$ is a numerical representation of $\succsim_i : Z_i \times B_i$

$\llcorner$ This is the approach that Ellickson et. al. takes.

$\boxed{2}$ Cole, Prescott - embraced the idea of lotteries.

$L(Z_i \times B_i) = \{ \gamma : \gamma_{(z,b)} \geq 0, \sum_{(z,b)} \gamma_{(z,b)} = 1 \}$

- This is the commodity space they used.

$L_i \equiv L(Z_i \times B_i) \neq Z_i \times Prob[B_i]$, but they are similar.

With $L(Z_i \times B_i)$, preferences are linear in probabilities.

- Here, $v_i \,\hat{=}\, \succsim_i (L_i)$ - preferences for mixtures are defined whereas in $\boxed{1}$, they are not

In the [2] interpretation, you may improve utility by introducing lotteries. (you are essentially adding new commodities)

The ordinary notion of Pareto optimality is in terms of $\succsim_i (Z_i \times B_i)$, but we can have Pareto improvements with respect to $\succsim_i' (L_i)$ subset of $Z_i \times \text{Prob}[B_i]$

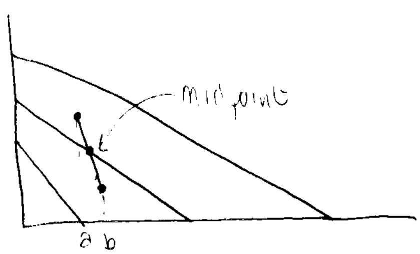
Define: $R_i(z,b) \equiv \{(z', \delta_{b'}) : v_i(z',b') \geq v_i(z,b)\}$ (at least as good as set)

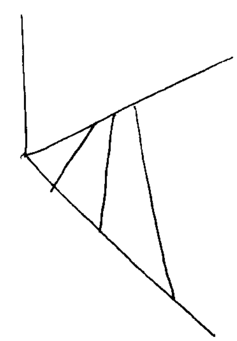
co $R_i(z,b) = \hat{R}_i(z,b) \equiv \{\sum \eta_{(z',b')} \delta_{(z',b')} = (w,\beta) : (z',b') \in R_i(z,b) +$ usual restrictions$\}$

Define $\tilde{v}_i(w,\beta) = \max\{ \sum \eta_{(z',b')} v_i(z',b') : \sum \eta_{(z',b')} (z', \delta_{b'}) = (w,\beta),$
$(w,\beta) \in \hat{R}_i(z,b)$, $(z',b') \in R_i(z,b)$ for $\eta_{(z',b')} > 0\}$

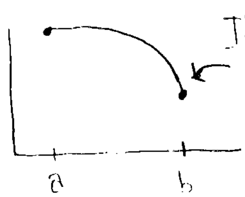
Note $\hat{v}_i(w,\beta) \geq \tilde{v}_i(w,\beta)$

Ostroy tried to show that $\hat{v}_i(w,\beta) = \tilde{v}_i(w,\beta)$, but it turns out that this is impossible. There is a well-known counterexample.

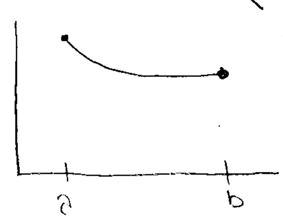




In the third dimension, is there a concave function which has these level curves?



If it looks like this, we are okay



This is what they really look like though, so we are in trouble.

$v_i^*(p, P_i) = \max_{(z,b)} \{ v_i(z,b) - pz + P_i(b)\}$

$v_i^*(p, P_i | L) = \max\{ \sum \eta_{(z,b)} v_i(z,b) - \underbrace{\qquad\qquad}_{\text{tricky notation}} \}$

Suppose $B_i(p, P_i) = \{ \gamma : \langle (p, P_i), \gamma \rangle = 0 \}$

L set of lotteries I can buy if the money commodity disappeared

$V_i^*(p, P_i; 0) = \max \{ V_i(z, b) : \underbrace{-pz + P_i(b)}_{m_i} = 0 \}$

L deterministic budget constraint

L ordinal case

In the lottery case, you maximize:

$V_i^*(p, P_i | L)$ s.t. $B_i(p, P_i)$