Let $x \in M[\mathbb{R}^\ell]$ $\quad x = \sum \alpha_k \delta_{z_k}$ (measure w/finite support)

(F) $\quad \sum_k z_k x(z_k) = \sum_k z_k(\alpha_k) = 0$

$(\Leftrightarrow \int z\,dx = 0)$

Let $A(x) := \{ \Delta z : \sum n_k z_k \,;\, z_k \in \mathrm{supp}(x),\ n_k \in \{0,1,2,\ldots\}\}$

(†) $A(x) \cap \mathbb{R}^\ell_+ \setminus \{0\} = \emptyset \Rightarrow$ No arbitrage condition.
L necessary for equilibrium

Suppose $\Delta z \in A(x) \cap \mathbb{R}^\ell_+ \setminus \{0\}$

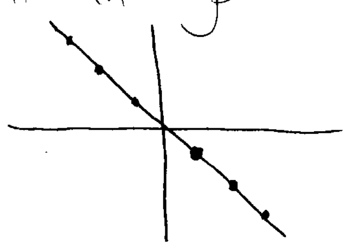
- $A(x) + A(x) = A(x)$

$\Rightarrow \lim_{k\to\infty} k \Delta z = \infty$ in at least one dimension.

Thm: If $A(x) \cap \mathbb{R}^\ell_+ \setminus \{0\} = \emptyset$, $\exists\, p$ s.t. $pA(x) \leq 0$

Since $z \in \mathrm{supp}(x) \Rightarrow -z = -\Delta z \in A(x) \Rightarrow -pz \leq 0$. If we assume $\sum_k z_k x(z_k) = 0$, then $pz = 0 \;\forall z \in \mathrm{supp}(x)$.

Trades will always be along this line:



We will assume that $\mathrm{supp}(x)$ is s.t. $\forall c = 1, \ldots, \ell \;\exists z \in \mathrm{supp}(x)$ s.t. $z = (z_1, \ldots, z_c, \ldots, z_\ell)$ has $z_c \neq 0$. Call commodities are traded.

This is nothing other than the canonical separating hyperplane theorem.