Knowledge and The Wealth of Nations - David Worsh

Conflict b/A pinhead factory and the invisible hand.

- conflict b/A increasing returns to scale and competition

Look up Liapunov's theorem

Let $I=[0,1]$

$z: I \to \mathbb{R}^\ell$ trade vector

$\int_I z \, d\lambda = 0 \Rightarrow z$ is feasible $(z_i \in Z_i \ \forall i)$

Liapunov: $\forall \varepsilon > 0 \ \exists \ \underset{\text{partition}}{\Pi} = \{E_k\}$ of I with $\lambda(E_k) < \varepsilon$ and

$\int_{E_k} z \, d\lambda = 0 \ \forall k.$

Krugman: This is not what Smith has in mind

Revolutions
- international trade (Krugman)
- growth (Romer)
- economic geography (Krugman)

Each of these three revolutions were driven by applications

Suppose $I = \{1, \dots, n\}$

$z: I \to \mathbb{R}$

$\sum_{i=1}^{n} z(i) = 0$

The analog to Liapunov is $z(i)=0 \ \forall i$. But the smallest partition we have is $F \subset I$ and $\sum_{i \in I} z(i) = 0$.

└ This picks up complementarity (in the continuum case, there is massive amounts of redundancy.)

Suppose $v_i : \mathbb{R}^\ell \to \mathbb{R} \cup \{-\infty\}$ and that $\ell = n$

$$v_I(0) = \max \left\{ \sum_{i=1}^{n} v_i(z_i) : \sum z_i = 0 \right\}$$

These models changed our view to:

$I = [0,1]$

$z(i) \in Z(i) \ \forall i, \quad z : I \to X$

Prefs are: $\left[\sum_{k=1}^{\infty} x_k^{1/2} \right]^2$ (Krugman) based on Dixit-Stiglitz

Continuum: $\left[\int_{[0,\infty)} x^{1/2} \, d\mu \right]^2$ (Romer)

Recall: $\mu = \sum \alpha_k \delta_{x_k} \in M[0,1]$

In the total variation norm,

$x_k \to x$ but $\| \delta_{x_k} - \delta_x \| = 2$

Weak* convergence Let $f : [0,1) \to \mathbb{R}$ be continuous.

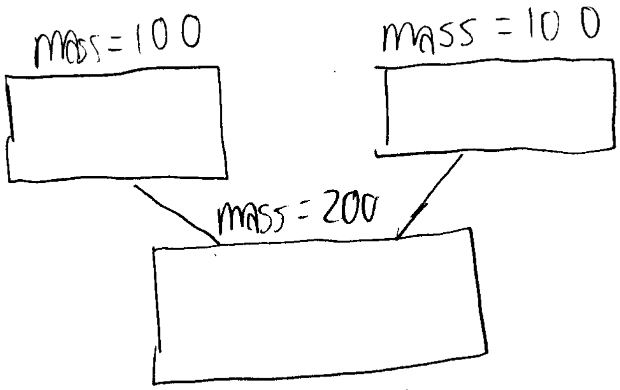
- $\int f \, \delta_{x_k} \to \int f \, \delta_x$

  $f(x_k) \longrightarrow f(x)$

- Here, a finite type economy is "like" a continuum type economy.

Which topology are we putting on X?

If we put weak* topology, then even if we have an infinite number of commodities, it is approximately as if we have a finite number of commodities



If we have weak* convergence, we do not have superadditivity in production.

You need fixed costs in order to keep $\left[\sum_{k=1}^{\infty} x_k^{1/2}\right]^2$ bounded.

Only thing holding back the economy in these examples is finite population

To exploit economies of scale, you need specialization.

- Growth, international trade, and large cities all depend on "being limited only by the extent of the market."

Let $x = (x_0, x_1, \ldots, x_n)$ be an allocation and p, P be prices, and let $r = (r_1, \ldots, r_n)$ be the population

- $x_0 \in M[B_0]$, $B_0 = \{(y,b) : y \in Y(b)\}$, $b = (T, s)$
- $x_i \in M[B_i]$, $B_i = \{(z,b) : z \in Z_i, b \in B_i\}$
- $p \in \mathbb{R}^l$, $P = (P_1, \ldots, P_n)$, $P_i : B_i \to \mathbb{R}$

$$\sum_{i \in T(b)} P_i(b) \geq py \quad y \in Y(b)$$

$[x, (p, P)]$ is an equilibrium if x is feasible and

- $x_i(z,b) > 0 \Rightarrow v_i(z,b) - pz + P_i(b) = v_i^*(p,P)$
- $x_0(y,b) > 0 \Rightarrow \sum_{i \in T(b)} P_i(b) = py$

This is too complicated as a notion of price-taking equilibrium. How do we arrive at equilibrium?

arbitrage: three types

(I) • Elementary $\longrightarrow p$

(II) • Commodity $\longrightarrow$ utility maximization wrt commodities

(III) • Entrepreneurial $\longrightarrow$ get from $x_i^m(z,b,m) > 0$ to P_i

• ie we see $m = -pz + \underbrace{Q_i(b)}_{\text{must be what ppl get paid}}$

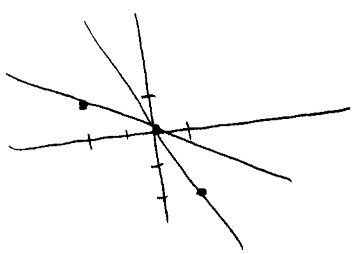
• it turns out we will have $Q_i(b) = P_i(b)$

Contrast arbitrage with core bargaining.

$x_i^m(z,b,m)$ and $x_0^m(y,b,m)$

Suppose we observe

$\begin{pmatrix} z \\ m \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} z' \\ m' \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$



$A^E(x^m)$ - elementary arbitrage opportunities associated with the trade x^m

$$A^E(x^m) = -\bigcup \left\{ (\Delta z, \Delta m) = \sum_i \sum_{(z,b,m)} (z,m)\, x_i^*(z,b,m) + \sum (y,m)\, x_0^*(y,b,m) \right\}$$

$x^* \in [x^m]^*$ = set of measures w/same support as x_i^m but taking on integer values.

↳ If $x_i^m(z,b,m) > 0$, $(x_i^m)^*(z,b,m) \in \{0,1,2,\dots\}$

In equilibrium, $A^E(x^m) \cap \mathbb{R}_+^\ell \times \mathbb{R}_{++}^\ell = \emptyset$ (necessary condition)

If $A^E(x^m) \cap \mathbb{R}_+^\ell \times \mathbb{R}_{++}^\ell = \emptyset \implies \text{cone}(A^E(x^m)) \cap \mathbb{R}_+^\ell \times \mathbb{R}_{++}^\ell = \emptyset$, you can back out prices.