$\max\{c\cdot x : Ax \leq b,\ x \geq 0\}$ (P)

$\min\{y\cdot b : yA \geq c,\ y \geq 0\}$ (D)

Let $\underline{X}(A,b,c)$ be the set of solutions to (P)

$\underline{Y}(A,b,c)$ be the set of solutions to (D)

- These can be empty.

$\mathbb{R}^n \supseteq \underline{X}$ is a correspondence which is convex and compact valued.

$\mathbb{R}^m \supseteq \underline{Y}$ is as well.

Is $\underline{Y}(A,\cdot,c)$ continuous?

What does $\underline{Y}(A,b_k,c)$, $b_k \to b$ look like?

Does $\underline{Y}(A,b_k,c) \to \underline{Y}(A,b,c)$?

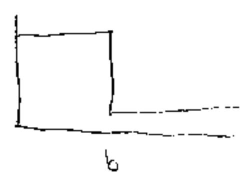
$$d(A,B) = \max_{a\in A}\ \min_{b\in B} d(a,b) + \max_{b\in B}\ \min_{a\in A} d(a,b)$$

$$= \max_{a\in A} d(a,B) + \min_{b\in B} d(A,b)$$

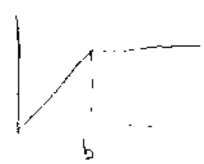
- Hausdorff difference is the metric we will use

Then we ask, does $d(\underline{Y}(A,b_k,c), \underline{Y}(A,b,c)) \to 0$?

It turns out that $\partial h(b) = \underline{Y}(A,b,c)$. This may look something like



corresponding to



This correspondence is UHC.

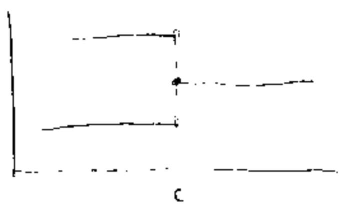
Def: $\underline{Y}(A,b_k,c)$ is UHC if $y_k \in \underline{Y}(A,b_k,c)$, $b_k \to b$ and $y_k \to y \Rightarrow y \in \underline{Y}(A,b,c)$

$\underline{Y}(A,\cdot,c)$ is UHC
$\underline{X}(A,b,\cdot)$ is UHC for the exact same reason.

Let f be concave. Define $\partial f(x) = \{p : f(x) - px \geq f(x') - px' \;\forall x'\}$
Let f be convex. Define $\partial f(x) = \{p : f(x) - px \leq f(x') - px' \;\forall x'\}$
$= \partial(-f(x))$.

How about $\underline{Y}(A,b,\cdot)$?
It turns out that $\overline{Y}(A,b,\cdot)$ and $\overline{X}(A,\cdot,c)$ are LHC

Defn: $Y(A,b,c)$ is LHC if $\forall y \in Y(A,b,c)$, whenever
$c_n \to c$ $\exists\, y_n \in Y(A,b,c_n)$ s.t. $y_n \to y$.
a fcn which is LHC and not UHC:



If we can find a solution to the QL model
such that $pz_i = 0$ $\forall i$ then we didn't need
money anyway.
We can premultiply v_i by λ_i to manipulate the
marginal utility of money compared to other commodities,
these v_is are analogous to C.

Virtual price equilibrium

$p \in \mathbb{R}^l$ prices of commodities
$b = (T,s)$, $s \in S(T)$. Define $T(b)$ to be the people in b.
$P_i(b)$ is payment received by $i \in T(b)$
$P_i(b): B_i \to \mathbb{R}$. $B_i = \{(T,s): T \ni i, s \in S(T)\}$

$P = (P_1, \ldots, P_n)$ is the set of contractual prices
The price system is the pair (p, P)

a <u>virtual price equilibrium</u> is a feasible allocation
$x = (x_0, x_1, \ldots, x_n)$, where $x_0 \in M[A_0]$, and
$A_0 = \{(y,b) : y \in \underbrace{Y(b)}_{= Y(T,s)}, b \in B_0\}$, $B_0 = \bigcup_{i \in I} B_i$

$x_i \in M[A_i]$ where $A_i = \{(z,b) : z \in Z_i(b), b \in B_i\}$. Feasibility is defined by

- $\sum_{(z,b)} x_i(z,b) = r_i$
- $\underbrace{\sum_i \left[\sum_{(z,b) \in A_i} z x_i(z,b) \right]}_{\text{excess demand}} - \underbrace{\sum_{b \in B_0} y x_0(y,b)}_{\text{aggregate production}} = 0$
- $\sum_z x_i(z,b) - \sum_y x_0(y,b) = 0 \quad \forall b \in B_i \quad \forall i$
  - ie $\forall i, j \in T(b)$, $\sum_z x_i(z,b) = \sum_z x_j(z,b)$

In addition, there's a pricing system (p, P)

- $(p,P)(y,b) = \underbrace{p \cdot y}_{\text{commodity profits}} - \underbrace{\sum_{i \in T(b)} P_i(b)}_{\text{contractual profits}} \underset{\text{by "CRS"}}{=} 0$ (feasibility of (p,P))

- In equilibrium, $P_i(b)$ is a profit share of $p \cdot y$.
- Here, the profit share are not exogenously determined

<u>Defn</u> a VPE is a vector $(x, (p,P))$ satisfying

1. x is feasible, (p,P) is feasible
2. $x_i(z,b) > 0 \Rightarrow v_i(z,b) - pz + P_i(b) = v_i^*(p, P_i)$
3. $x_0(y,b) > 0 \Rightarrow py - \sum_{i \in T(b)} P_i(b) = 0$

The dual is: $\min \sum_{i \in I} r_i q_i$

$$\text{s.t. } q_i + pz - P_i(b) \geq v_i(z,b) \quad \forall i \quad \forall (z,b) \in Z$$

$$\sum P_i(b) - py \geq 0$$

$$\Rightarrow q_i \geq v_i(z,b) - pz + P_i(b)$$

$$\Rightarrow q_i = r_i^*(p, P_i) \text{ is optimal}$$

What is a better way of thinking about equilibrium in light of so many prices?

- arbitrage will be the key.