Market clearing conditions

1] $\sum_{(z,b)\in Z_i\times B_i} x_i(z,b) = r_i$

2] $\sum_i \Big[\underbrace{\sum_{(z,b)} z\, x_i(z,b)}_{\approx \text{Expected trade for type } i}\Big] - \underbrace{\sum_{b\in B} y(b)\, x_0(b)}_{\approx \text{Expected production}} = 0$

3] $\sum_z x_i(z,b) - x_0(b) = 0 \quad \forall b \in B_i \ \forall i \in T(b)$

What are the implications in terms of idealized pricing?

From 2], we will get $p \in \mathbb{R}^\ell$ (commodity prices)

For 3], let $P_i : B_i \to \mathbb{R} \Rightarrow P = (P_1, \ldots, P_n)$ (contractual prices)

- This a very complicated pricing system
- Contrast this with the parsimony of prices in 2]
  - The parsimony of prices in 2] represents the decentralization role of prices

$[\overline{X = (x_0, x_1, \ldots, x_n)}, (p, P)]$ price-taking equilibrium.

Looking at $x_i(z,b) > 0$ (ie the support of x)

There are far more prices in P than elements of the support of x.

Each individual looks at (p, P_i) to

$\max\{V_i(z,b) + m : m + pz = \underbrace{P_i(b)}_{\text{amount } i \text{ is given for choosing } b}, (z,b) \in Z_i \times B_i\}$

$= \max\{V_i(z,b) - pz + P_i(b) : (z,b) \in Z_i \times B_i\} = V_i^*(p, P_i)$

- Prices will tell you which team, which action, and which net trade to carry out.
- Prices are telling you what not to do.
- Formally, for complete markets, you must have (p, P).

- To say that we would actually need to rely on (p,P) would be an embarassment.

Competition }
Decentralization } Decentralization is not what is going on in this model.

$v_i : Z_i \times B_i \to \mathbb{R}$
$z_i \in \mathbb{R}^l$

If $v_i : Z_i \to \mathbb{R}$, then there is a well-defined concavification operation: $\hat{v}_i(\bar{z}) = \max\{\Sigma \lambda_k v_i(z_k),\ z_k \in Z_i,\ \Sigma \lambda_k z_k = \bar{z},\ \lambda_k \geq 0, \Sigma \lambda_k = 1\}$

$\hat{v}_i(\bar{z}) \geq v_i(\bar{z})$ if $\qquad \hat{v}_i : \hat{Z}_i \to \mathbb{R}$

1] Z_i is not convex
2] v_i is not concave.

How do we extend concavification to work for $v_i : Z_i \times B_i \to \mathbb{R}$?

Let $M[B_i]$ be the set of measures on B_i.
$M[B_i]$ is a linear space and
$M[B_i] \underset{\uparrow}{\approx} \mathbb{R}^{|B_i|}$
isomorphic

Let $\delta_b(E) = \begin{cases} 1 & \text{if } b \in E \\ 0 & \text{else} \end{cases}$

$\hat{B}_i = \left\{ \sum_{b \in B_i} \pi(b) \delta_b \ : \pi(b) \geq 0, \sum_{b \in B_i} \pi(b) = 1 \right\}$

= set of lotteries over indicator functions of points in B_i

$\hat{V}_i : \hat{Z}_i \times \hat{B}_i \to \mathbb{R}$

$$\hat{V}_i(\omega, \beta) = \max \left\{ \sum_{(z,b)} \gamma(z,b) V_i(z,b) : \sum_{(z,b)} \gamma(z,b) \cdot (z, \delta_b) = (\omega, \beta), \gamma(z,b) \geq 0, \sum_{(z,b)} \gamma(z,b) = 1 \right\}$$

$\omega \in \hat{Z}_i$, $\beta \in \hat{B}_i$