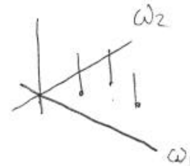
Firms would exist even if there were not opportunistic behavior. Opportunistic behavior is not the only reason. There must be some sort of coordination gains.

Let X, Y be two metric spaces with $\mathcal{B}(X)$ and $\mathcal{B}(Y)$ be the Borel sigma algebras.

Defn: a function $f: X \to Y$ is measurable if $\forall$ $E \in \mathcal{B}(Y)$, $f^{-1}(E) = \{x : f(x) \in E\} \in \mathcal{B}(X)$.

Let $X = [0,1]$ be an index set for consumers. $\mathcal{B}([0,1])$
$f: [0,1] \to \mathbb{R}^{\ell}$ be, say, a description of initial endowments.

Let $\mu \in \underbrace{M(\mathbb{R}^{\ell})}_{\text{set of measures on } \mathbb{R}^{\ell}}$.



Measure with finite support.

Note here that there are no names. Does μ capture the relevant component of the description of the problem.

$\mu(E) = \lambda \circ f^{-1}(E)$ where λ is the Lebesgue measure on $[0,1]$.

The first descriptions of continuum economies used index sets. Now, we characterize economies using measures.
Where possible, we will restrict our attention to measures with finite support.

Suppose $f: X \to Y$ where $Y = \underbrace{\mathcal{U}}_{\text{set of u-fns}} \times \mathbb{R}^{\ell} \times \mathbb{R}^{\ell}$ $(u, \omega, z) \in Y$

[Hildenbrand, Mas-Collel] How to define a metric?

Let $u, u' \in \mathcal{U}$ $d_{K^n}(u,u') = \max\{|u(z) - u'(z)| : z \in \underbrace{K^n}_{\text{compact}} \subset \mathbb{R}^{\ell}\}$

$K^{n+1} \supseteq K^n \supseteq \ldots$ increasing sequence of, say, cubes.

$d(u,u') = \sum_{n=1}^{\infty} \frac{1}{2^n} d_{K^n}(u,u')$ defines a metric on $\mathcal{U}$,

a set is compact if it can be guarded by finitely many guardians with arbitrary short-sightedness. (can approximate arbitrarily well)

<u>If</u> we assume $\mathcal{U}$ to be compact, we can use a finite grid.

Let $I = \{1, \dots, n\}$. $V_i : Z_i \times B_i \to \mathbb{R}$. $V_i(\overset{\text{does a trade}}{z}, \overset{\text{participates in a team activity}}{b})$

$Z_i \subseteq \mathbb{R}$ set of feasible trades

$B_i = \{ b = (T, S, \overset{y(b)}{y}) : T \in \mathbb{T}_i, s \in S(T), y \in \overset{\subseteq \mathbb{R}^{\ell}}{Y(T,S)} \}$

└ generalization of the commodity space.

B_i is not a subset of a linear space, in general.

└ There is no algebraic structure here.

Prices are linear functions on the commodity space, but we don't have a linear space, so what do we do?

Two things to highlight

- Feasible allocation? How do we describe one?

$\mathcal{E} = \{ (Z_i \times B_i), (V_i) \}$, $\underset{\text{population}}{r} = (r_1, \dots, \underset{\text{mass of people of type } n}{r_n})$

$\mathcal{E}_r = \{ (Z_i \times B_i), (V_i), r \}$

There are substantial indivisibilities here.

Use $U_i(z, b, m) = V_i(z, b) + m$

Let $M[Z_i \times B_i]$ be the set of positive measures of finite support.

$X_i \in M[Z_i \times B_i]$

(1) $\sum_{(z,b)\in Z_i \times B_i} x_i(z,b) = r_i \quad \forall i$ (people clearing condition)

Let $x_0 \in M[B]$ where $B = \cup_i B_i$

Notation: $z = \begin{pmatrix} + & \text{buy} \\ - & \text{selling} \end{pmatrix}$

$y = \begin{pmatrix} + & \text{selling} \\ - & \text{buying} \end{pmatrix}$

(2) $\sum_i \sum_b z\, x_i(z,b) - \underbrace{\sum_{b\in B} \underbrace{y(b)}_{\substack{\text{amout} \\ \text{produced} \\ \text{by team } b}} \underbrace{x_0(b)}_{\substack{\text{mass of teams} \\ \text{engaging in activity } b}}}_{\text{aggregate buying/selling by teams}} = 0 \in \mathbb{R}^\ell$ (commodity clearing)

How do we ensure that teams are matching up?

(3) $x_0(b) - \sum_z x_i(z,b) = 0 \in \mathbb{R} \quad \forall b \in B_i, \forall i$ (team clearing)

The assignment model is a special case of this.

$$\sum_z x_i(z,b) = \sum_z x_{i'}(z,b) \quad \forall i, i' \in T(b)$$

(3) is a much finer form of matching than (2).

This is a more difficult resource allocation problem than the standard Neoclassical model.