

Review Question for Final Exam

Question 1:

Suppose that you are given with the random sample y_1, \dots, y_n from a distribution whose density function is given by

$$f(y) = \frac{2}{\theta^2} (\theta - y), \quad \text{for } 0 < y < \theta.$$

1. Provide the MLE for θ , say $\hat{\theta}_n$.
2. Provide the asymptotic distribution for $\hat{\theta}_n$.
3. Is the estimator in (1) unbiased estimator for θ ? Justify your answer.
4. Provide a test statistic for testing the null hypothesis $H_0: \theta^2 = 1$ against the hypothesis $H_1: \theta^2 < 1$.

Question 2:

Consider the following two equations:

$$E[y_i | x_i] = x_i' \beta \tag{12.1}$$

$$E^*[y_i | x_i] = x_i' \beta^l, \tag{12.2}$$

for $i = 1, \dots, n$. Note that equation (3) states that the true conditional expectation of y_i , conditional on x_i , is linear, while equation (4) characterizes the linear *predictor* of y_i conditional on x_i . Let $\hat{\beta}$ and $\hat{\beta}^l$, be the least-squares estimators of β and β^l , respectively. Define

$$\begin{aligned} \alpha_n &= \sqrt{n}(\hat{\beta} - \beta), \quad \text{and} \\ \alpha_n^l &= \sqrt{n}(\hat{\beta}^l - \beta^l). \end{aligned}$$

1. Derive the asymptotic distribution of α_n .
2. Derive the asymptotic distribution of α_n^l .
3. Under what conditions will α_n and α_n^l have the same asymptotic distribution.

Question 3:

Consider the linear model

$$y_i = x_i' \beta + u_i,$$

where

$$\begin{aligned} u_i &= \sigma(x_i) \varepsilon_i, \\ \sigma(x_i) &= 1 + x_i' \gamma, \quad \text{and} \\ \varepsilon_i | x_i &\sim \text{i.i.d. } N(0, \sigma_\varepsilon^2). \end{aligned}$$

1. Provide an estimator for β and show that the suggested estimator is consistent.
2. Show that the estimator suggested in (1) is asymptotically normal, and provide its asymptotic covariance matrix.
3. Discuss whether or not one can estimate γ . If your answer is yes provide the details of the proposed estimator. If the answer is no discuss in details why not.

Question 4:

Suppose we have a model of the form

$$\begin{aligned} y_i &= \alpha_0 \exp \{x_i' \beta_0\} + \varepsilon_i, \quad \text{where} \\ E(\varepsilon_i | x_i) &= 0, \end{aligned}$$

for $i = 1, \dots, n$, where β_0 is a $K \times 1$ vector of parameters and α_0 is a scalar parameter.

1. Under what condition is α_0 identified?
2. Assume that the condition(s) in (1) is (are) satisfied. Suggest a $K \times 1$ moment function $\varphi_1(y_i, x_i, \alpha, \beta)$ such that

$$E[\varphi_1(y_i, x_i, \alpha_0, \beta_0)] = 0.$$

3. Suggest an additional $K \times 1$ moment function $\varphi_2(y_i, x_i, \alpha, \beta)$ such that

$$E[\varphi_2(y_i, x_i, \alpha_0, \beta_0)] = 0.$$

4. Show how to combine the moment functions in (2) and (3) to get the most efficient estimators for $\theta_0 = (\alpha_0, \beta_0)'$, say $\hat{\theta}_n$.
5. Provide the asymptotic distribution function for the estimator for $\hat{\theta}_n$ obtained in (4).
6. Suggest a consistent estimator for the asymptotic covariance matrix derived in (5).

Question 5:

Consider the two equation model given by

$$\begin{aligned} y_{1i} &= x_{1i}' \beta_1 + u_{1i}, \\ y_{2i} &= x_{2i}' \beta_2 + u_{2i}, \end{aligned}$$

where x_{1i} and x_{2i} are $K_1 \times 1$ and $K_2 \times 1$ vectors of regressors, respectively, β_1 and β_2 are the corresponding unknown vectors of parameters. Let $u_i = (u_{1i}, u_{2i})'$, and assume that

$$u_i | x_{1i}, x_{2i} \sim \text{i.i.d. } (0, \Sigma_u).$$

1. Describe how to obtain the most efficient estimates for β_1 and β_2 .
2. Provide a consistent estimate for Σ_u .
3. Suppose that $\text{Cov}(u_{1i}, u_{2i}) = 0$. How would it change the answer in (1)?

4. Suppose that $x_{1i} = x_{2i}$ for all $i = 1, \dots, n$. How would it change the answer in (1)?

Question 6:

Consider the two equation model given by

$$\begin{aligned} y_{1i} &= \gamma_1 y_{2i} + x'_{1i} \beta_1 + u_{1i}, \\ y_{2i} &= \gamma_2 y_{1i} + x'_{2i} \beta_2 + u_{2i}, \end{aligned}$$

where x_{1i} and x_{2i} are $K_1 \times 1$ and $K_2 \times 1$ vectors of regressors, respectively, β_1 and β_2 are the corresponding unknown vectors of parameters, and γ_1 and γ_2 are two scalar parameters. Let $u_i = (u_{1i}, u_{2i})'$, and assume that

$$u_i | x_{1i}, x_{2i} \sim \text{i.i.d. } (0, \Sigma_u).$$

1. Describe in details the conditions under which the model's parameters are identified.
2. Assume that the conditions in (1) are satisfied. Provide the most efficient estimates for $\theta_1 = (\gamma_1, \beta_1)'$, say $\hat{\theta}_1$.
3. Derive the asymptotic distribution for $\hat{\theta}_1$.
4. Suggest a test for the hypothesis $H_0: \gamma_1 = \gamma_2$ against $H_1: \gamma_1 \neq \gamma_2$.

Question 7:

Consider the binary discrete choice model given by

$$\Pr(y_i = 0) = \frac{\exp(x'_i \gamma)}{1 + \exp(x'_i \gamma)},$$

for $i = 1, \dots, n$.

1. Provide the MLE for γ , say $\hat{\gamma}_n$.
2. Show that the MLE can be considered as an MM estimator.
3. Compute the asymptotic covariance for $\hat{\gamma}_n$ and provide a consistent estimate for the asymptotic covariance. Justify your answer.

Question 8:

Consider the linear model

$$\begin{aligned} y_i &= x'_i \beta + u_i, \quad \text{where} \\ E(u_i | x_i) &\neq 0, \end{aligned}$$

for $i = 1, \dots, n$, where x_i is a $K \times 1$ vector of regressors. Suppose that there exist a vector of random variables z_i such that

$$E(u_i | z_i) = 0,$$

where z_i is an $M \times 1$ vector, with $M > K$.

1. Show that a least-squares regression of y_i on x_i will yield an inconsistent estimator for β .
2. Suggest an Instrumental variable estimator for β using the entire vector of instruments z_i .
3. Show that the estimator suggested in (2) can be viewed as a GMM estimator.
4. Using the fact established in (3), provide the asymptotic distribution of the estimator for β .
5. Provide a consistent estimator for the asymptotic covariance matrix established in (4). Justify your answer.

Question 9:

Consider the system of equation model given by

$$\begin{aligned} y_{1i} &= \gamma_{12}y_{2i} + \gamma_{13}y_{3i} + x'_{1i}\beta_1 + u_{1i}, \\ y_{2i} &= \gamma_{23}y_{3i} + x'_{2i}\beta_2 + u_{2i}, \\ y_{3i} &= x'_{3i}\beta_3 + u_{3i}, \end{aligned}$$

where x_{1i} , x_{2i} , and x_{3i} are $K_1 \times 1$, $K_2 \times 1$, and $K_3 \times 1$, vectors of exogenous regressors. Let $u_i = (u_{1i}, u_{2i}, u_{3i})'$ and assume that

$$u_i \sim \text{i.i.d. } (0, \Sigma_u).$$

1. Assume that $\Sigma_u = \text{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2)$. Determine which of the above parameters is identified.
2. How would your answer to (1) change if the off-diagonal terms of Σ_u are not zero? Explain briefly.
3. Provide estimates for all the parameters that you determine in (1) to be identified.
4. Provide a consistent estimate for Σ_u .

Question 10:

Let the system of equation model be given by

$$\begin{aligned} y_{1i} &= \gamma_{12}y_{2i} + x'_{1i}\beta_1 + u_{1i}, \\ y_{2i} &= \gamma_{23}y_{3i} + x'_{2i}\beta_2 + u_{2i}, \\ y_{3i} &= \gamma_{31}y_{1i} + x'_{3i}\beta_3 + u_{3i}, \end{aligned}$$

where x_{1i} , x_{2i} , and x_{3i} are $K_1 \times 1$, $K_2 \times 1$, and $K_3 \times 1$, vectors of exogenous regressors.

1. Express this system in the concise form

$$y'_i \Gamma = x'_i B + u'_i,$$

where y_i is a $J \times 1$ vector, Γ is a $J \times J$ matrix of coefficients, and B is a $K \times J$ matrix of coefficients.

2. Verify whether or not that order condition for identification holds for the above system of equation.
3. Provide the reduced-form of the model and explain how to estimate efficiently the reduced form parameters.
4. Suggest a test for the null hypothesis $H_0: \gamma_{12} = \gamma_{23} = \gamma_{31}$, against $H_1: \text{Not } H_0$.