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Economics 203C
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Problem Set 6

Due: Wednesday May 28, 2003

Question 1:

In this exercise we will conduct a Monte Carlo experiment that is designed to examine the *small sample* properties of the three tests discussed in Lecture Note 11, namely the Wald, Lagrange multipliers (LM), and likelihood ratio (LR) tests.

Consider the linear regression model given by

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \beta_4 x_{4i} + \varepsilon_i$$

Let $x_i^0 = (x_{2i}, x_{3i}, x_{4i})'$ and it is given that

$$x_i^0 \sim N(\mu_x, \Omega_x),$$

where

$$\begin{aligned} \mu_x &= (2,2,3)', \quad \text{and} \\ \Omega_x &= \begin{pmatrix} 4.456 & -0.274 & 0.227 \\ -0.274 & 5.323 & 0.017 \\ 0.227 & 0.017 & 5.247 \end{pmatrix}. \end{aligned}$$

The value for the parameter vector β is given by:

$$\beta = (\beta_1, \beta_2, \beta_3, \beta_4)' = (1, .5, -.5, .25,)'.$$

In this exercise we draw 500 samples. A draw of an observation in a sample is constructed as follows

$$x_{d_1 i} = \mu_x + P\xi_i,$$

where P is such that $PP' = \Omega_x$, and ξ_i is a draw from a standard normal distribution. Then,

$$y_{di} = x'_{di}\beta + u_i,$$

where u_i is a draw from a student t distribution with 5 degrees of freedom and $x_{di} = \left(1, x'_{d_1 i}\right)'$.

Draw 500 samples each of 100 observations.

For each sample do the following:

1. Compute the optimal GMM estimator for β, say $\widehat{\beta}_n$, based on the moment conditions given by

$$\varphi(y_i, x_i, \beta) = (y_i - x_i'\beta)\, z_i,$$

where

$$z_i' = \left(1, x_{2i}, x_{3i}, x_{4i}, x_{2i}^2, x_{3i}^2, x_{4i}^2\right)'.$$

2. Compute a consistent estimator for the asymptotic covariance of the optimal GMM estimate.

3. Construct the Wald, LM, LR test statistics for the hypothesis

$$\text{H}_0\text{: } r(\beta) = \beta_2\beta_3 + \beta_4 = 0.$$

4. Store (for later use) the three test statistics obtained in (3).

5. Once you are done with (1) through (4) for each of the 500 samples drawn, plot a histogram for each of the 500 Wald, LM, and LR statistics obtained for the 500 samples.

6. Discuss briefly the results obtained in (5). In particular compute the actual (empirical) type I error that one would get for each of the above three statistics.

Question 2:

Show that any maximum likelihood estimator (MLE) is, in fact, a GMM estimator.

Question 3:

Consider the model given by

$$\begin{aligned} y_i &= g(x_i;\theta_0) + u_i, \\ E(u_i|x_i) &= 0, \end{aligned}$$

for $i = 1, ..., n$, where the parameter vector $\theta_0 \in \Theta \subset R^K$, x_i is a $K \times 1$ vector of regressors, and $g(\cdot)$ is a known non-linear function.

1. Show that the population parameter vector is obtained as a solution to

$$\min_{\theta\in\Theta} E\left[(y_i - g(x_i;\theta))^2\right].$$

2. Provide the conditions that make the parameter vector θ_0 unique.

3. Provide the sample analog of the population moments from which one can obtain an estimator for θ_0. Denote these moment conditions by $\varphi_1(y, x; \theta)$.

4. Suggest additional K moment conditions to those in (3). Denote the additional moment conditions by $\varphi_2(y, x; \theta)$. Show that

$$E(\varphi_2(y, x; \theta_0)) = 0.$$

5. Suggest an optimal GMM estimator for θ_0 based on $\varphi_1(y, x; \theta)$ and $\varphi_2(y, x; \theta)$ from (3) and (4).

6. Provide the asymptotic distribution for the estimator suggested in (5).

7. Provide the Wald, LM, and LR test statistics for the null hypothesis

$$\text{H}_0\text{: } \sum_{k=1}^{K} \theta_{k0}^2 = 1.$$