Econ 201C: General Equilibrium and Welfare Economics

Marginal Product Inequality and PCE

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1 The Marginal Product Inequality

Throughout the last several weeks in class, we have repeatedly acknowledged that in a price-taking equilibrium, individuals provide weakly more for the economy than they take away from it. That is,

$$MP_i \geq v_i^* (p)$$

This claim went without proof, but it will go without proof no more.

Lemma 1 *Let $p \in \partial v_{-i} (-z_i)$. Then $SOC_{-i} (-z_i) \geq p (-z_i)$ for all i.*

Proof. Let i be arbitrary and let $p \in \partial v_{-i} (-z_i)$. By definition,

$$v_{-i} (-z_i) - p (-z_i) \geq v_{-i} (z) - pz \text{ for all } z$$

Let $z = 0$. Then

$$\begin{aligned} v_{-i} (-z_i) - p (-z_i) &\geq v_{-i} (0) \\ v_{-i} (-z_i) - v_{-i} (0) - p (-z_i) &\geq 0 \end{aligned}$$

Or

$$SOC_{-i} (-z_i) \geq p (-z_i)$$

Since i was arbitrary, this holds for all i, establishing the result. ■

Lemma 2 *$SOC_{-i} (-z_i) - p (-z_i) = MP_i - v_i^* (p)$ for all i.*

Proof. Let i be arbitrary

$$\begin{aligned} MP_i - v_i^* (p) &= v_I (0) - v_{-i} (0) - v_i^* (p) \\ &= v_I (0) - v_{-i} (0) - [v_i (z_i) - pz_i] \\ &= v_i (z_i) + v_{-i} (-z_i) - v_{-i} (0) - v_i (z_i) - p (-z_i) \\ &= v_{-i} (-z_i) - v_{-i} (0) - p (-z_i) \\ &= SOC_{-i} (-z_i) - p (-z_i) \end{aligned}$$

Since i was arbitrary, this holds for all i. ■

Proposition 3 *Let $[(z_i), p]$ be a price-taking equilibrium for $\mathbf{v} = (v_i)$. Then $MP_i \geq v_i^* (p)$ for all i.*

Proof. Since $[(z_i), p]$ is a price-taking equilibrium for $\mathbf{v}$, we necessarily have that $p \in \partial v_I (0)$, which is equivalent to $p \in \partial v_i (z_i)$ and $p \in \partial v_{-i} (-z_i)$. By lemma 1, this gives us the inequality $SOC_{-i} (-z_i) - p (-z_i) \geq 0$. By lemma 2, then, we have $MP_i - v_i^* (p) \geq 0$, which is the desired result. ■

It should be noted that this result does not always hold. When we consider economies in which there are negative externalities or public goods, the inequality may indeed go in the other direction.

2 Perfectly Competitive Equilibrium

It is often the case that the inequality established above holds strictly. As we will see later in the course, when this is true, there will be opportunities for individuals to manipulate the price system, which in general will lead to inefficiency. Most standard treatments of general equilibrium theory remedy this with the strong behavioral assumption of price-taking behavior. However, in this course, we wish to understand under which conditions this assumption is not necessary. Indeed, when such conditions hold, we will call the resulting equilibrium a perfectly competitive equilibrium.

There are three equivalent definitions of a perfectly competitive equilibrium in the quasilinear model. Lemma 2 above establishes the equivalence of two of the definitions. I will show the equivalence of all three. First, recall the definition of a price-taking equilibrium.

Definition 4 *A vector* $[(z_i^*, m_i^*), p^*]$ *is a* ***price-taking equilibrium*** *if*

1. $\sum_{i=1}^n z_i^* = 0, \ \sum_{i=1}^n m_i^* = 0$
2. $v_i^*(p^*) = v_i(z_i^*) - p^* \cdot z_i^*$

The three equivalent definitions of a perfectly competitive equilibrium are

Definition 5 *A vector* $[(z_i^*, m_i^*), p^*]$ *is a* ***perfectly competitive equilibrium*** *if it is a price-taking equilibrium and* $v_i^*(p^*) = MP_i$ *for all* i. *(Full appropriation)*

Definition 6 *A vector* $[(z_i^*, m_i^*), p^*]$ *is a* ***perfectly competitive equilibrium*** *if it is a price-taking equilibrium and* $SOC_{-i}(-z_i^*) = p^* \cdot (-z_i^*)$ *for all* i. *(Social cost equals private cost)*

Definition 7 *A vector* $[(z_i^*, m_i^*), p^*]$ *is a* ***perfectly competitive equilibrium*** *if it is a price-taking equilibrium and* $p^* \in \partial v_{-i}(0)$ *for all* i. *(PEDS)*

Remark 8 *The PEDS condition says that no individual has the capacity to change prices by leaving the economy.*

The equivalence of the "full appropriation" and the "social cost equals private cost" conditions was established in Lemma 2. (i.e. $v_i^*(p^*) = MP_i \iff SOC_{-i}(-z_i^*) = p^* \cdot (-z_i^*)$) I will now establish that "PEDS" is equivalent to "social cost equals private cost."

Proposition 9 *Let* $[(z_i^*, m_i^*), p^*]$ *be a price-taking equilibrium. Then* $p^* \in \partial v_{-i}(0)$ *for all* i *if and only if* $SOC_{-i}(-z_i^*) = p^* \cdot (-z_i^*)$ *for all* i.

Proof. Since $[(z_i^*, m_i^*), p^*]$ is a price-taking equilibrium, we will necessarily have that $p^* \in \partial v_{-i}(-z_i^*)$ or

$$v_{-i}(-z_i^*) - p^* \cdot (-z_i^*) \geq v_{-i}(z) - p^* \cdot z \text{ for all } z$$

In particular, letting $z = 0$,

$$v_{-i}(-z_i^*) - p^* \cdot (-z_i^*) \geq v_{-i}(0)$$

Next, we have that $p^* \in \partial v_{-i}(0)$ for all i if and only if

$$v_{-i}(0) - p^* \cdot (0) \geq v_{-i}(z) - p^* \cdot z \text{ for all } z$$

In particular, letting $z = -z_i^*$,

$$v_{-i}(0) \geq v_{-i}(-z_i^*) - p^* \cdot (-z_i^*)$$

This gives us

$$v_{-i}(-z_i^*) - p^* \cdot (-z_i^*) = v_{-i}(0)$$

Or

$$\begin{aligned} v_{-i}(-z_i^*) - v_{-i}(0) &= p^*(-z_i^*) \\ SOC_{-i}(-z_i^*) &= p^*(-z_i^*) \end{aligned}$$

Which is the desired equality. ■