Mechanism Design and $\exists$ A

$$\begin{tikzcd} V \arrow[rr, "F(v)"] \arrow[dr, "a^F(v{,}\mathcal{I})"'] & & Z \times \mathbb{R}^n \\ & A \arrow[ur, "f(v)"'] & \end{tikzcd}$$

$\mathcal{I} = (A_1, \dots, A_n; f)$

$f : A_1 \times \dots \times A_n \to Z \times \mathbb{R}^n$

$F(v) = f(a^F(v, \mathcal{I}))$

$a^F(v, \mathcal{I}) \to p(v) \to \pi_i^{\overbrace{WE}^{F}}(v) = v_i^*(p(v))$

$\pi_i(v) = v_i(z_i(v)) + m_i(v)$ ie: $F_i(v) = (z_i(v), m_i(v))$

Price-taking is a particular institutional arrangement.

Let us consider direct mechanisms $F : V_1 \times \dots \times V_n \to Z \times \mathbb{R}^n$

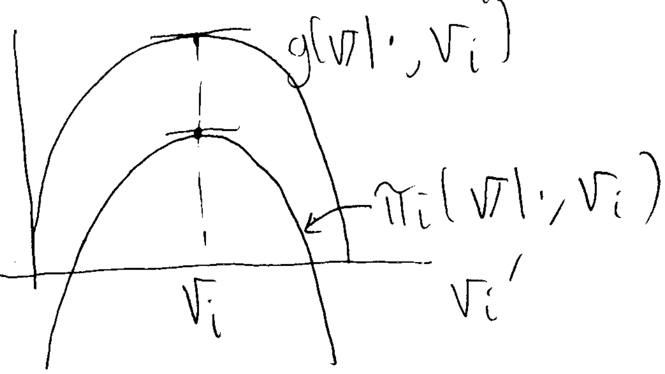
$\pi_i^F(v) = v_i(z_i(v)) + m_i(v)$

where $\sum_{i=1}^n v_i(z_i(v)) = \max\{\sum_{i=1}^n v_i(z_i) : (z_1, \dots, z_n) \in Z\}$

(ie F is <u>efficient with respect to the non-money commodity</u>)

Define $\pi_i^F(v | v_i'; v_i) = v_i(z_i(v | v_i')) + m_i(v | v_i')$

Suppose we make $\pi_i^F(v | v_i'; v_i) = g(v | v_i'; v_i) - h_i(v_{-i})$



Prop: $v_i = \underset{v_i' \in V_i}{\text{argmax}} \{ g(v|v_i'; v_i) \}$

$$g(v|v_i'; v_i) - h_{-i}(v_{-i}) = \sum_{j \neq i} v_j(z_j(v|v_i')) + v_i(z_i(v|v_i')) - h_i(v_{-i})$$

$$m_i(v|v_i') = \sum_{j \neq i} v_j(z_j(v|v_i')) - h_i(v_{-i})$$

It is possible to construct a non-manipulable mechanism that is efficient with respect to the non-money problem. But we will probably not have $\sum_{i=1}^{n} m_i(v|v_i') = 0$ (budget balancing.)

Let $F_i^h(v|v_i') = \left(z_j(v|v_i'), \sum_{j \neq i} v_j(z_j(v|v_i')) - h_i(v_{-i})\right)$

This class of mechanisms are non-manipulable and efficient wrt the non-money commodity

What if $\pi_i^F(v|v_i'; v_i) = \alpha_i g(v|v_i'; v_i)$? Then F is non-manipulable, efficient wrt non-money commodity and budget balancing. But

$$\begin{aligned}
\pi_i^F(v|v_i'; v_i) &= \alpha_i g(v|v_i'; v_i) \\
&= \alpha_i \left[\sum_{j \neq i} v_j(z_j(v|v_i')) + v_i(z_i(v|v_i')) \right] \\
&= v_i(z_i(v|v_i')) + m_i(v|v_i')
\end{aligned}$$

$$\Rightarrow m_i(v|v_i') = \alpha_i \sum_{j \neq i} v_j(z_j(v|v_i')) + (\alpha_i - 1) v_i(z_i(v|v_i'))$$

But this is not computable by the mechanism designer since it is a function of v_i, the true type of individual i, since this information is known only to individual i.

Recall in the OE model, the following definitions.

$$\Delta \pi_i(v; v_i') = \pi_i(v_{-i}, v_i') - \pi_i(v)$$

$$\Delta g(v; v_i') = g(v_{-i}, v_i') - g(v)$$

In the general OE model, we could not say what the sign of $\Delta g(v; v_i')$ was.

In the current situation, define

$$\Delta \pi_i(v; v_i') = \pi_i(v|v_i'; v_i) - \pi_i(v|v_i; v_i)$$

$$\Delta g(v; v_i') = g(v|v_i'; v_i) - g(v|v_i; v_i)$$

Recall: if $\Delta \pi_i(v; v_i') \cdot \Delta g(v; v_i') < 0$, then there are some incentive problems.

The (not very subtle) solution to this was full appropriation. $\Delta \pi_i(v; v_i') = \Delta g(v; v_i')$

$$\Rightarrow \Delta \pi_i(v; v_i') \; \Delta g(v; v_i') \geq 0$$

The coordination problem does not occur in the mechanism design framework, since each individual reveals so much information.

$$\overline{\pi}_i^F(v/v_i';v_i) = v_i(z_i(v/v_i)) + m_i(v/v_i')$$

with $m_i(v/v_i') = \sum_{j\neq i} v_j(z_j(v/v_i')) - h_i(v_{-i})$

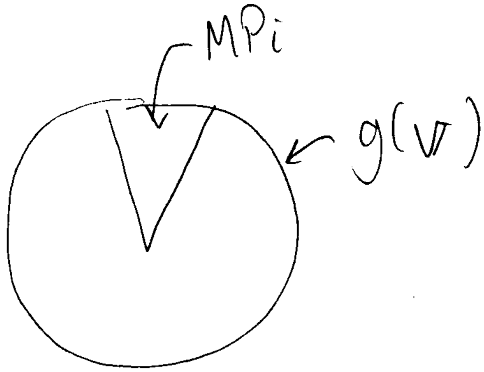
(ie i has to pay everything below the utility rights and gets to keep everything above the utility rights.)

Suppose $m_i(v/v_i') = \overbrace{v_{-i}(-z_i(v/v_i')) - v_{-i}(0)}^{h_{-i}(v_{-i})}$

$$= SOC_{-i}(z_i(v/v_i'))$$

Find functions $h_i(v_{-i})$ such that

- $\sum_{i=1}^{n} m_i(v/v_i') = \sum_{i=1}^{n}\sum_{j\neq i} v_j(v/v_i') - \sum_{i=1}^{n} h_i(v_{-i})$



Recall: $\sum MP_i > g(v)$

and $\sum MP_i \to g(v)$

are there any situations in which $\frac{g(v_1, \ldots, v_n)}{n} \uparrow +\infty$?

What if we have public goods?
If the public goods are costless, $\sum_{i=1}^{n} MP_i < g(v_T)$

- We have replica invariance here.
- as we replicate infinitely, $\lim_{k\to\infty} \sum_{i=1}^{kn} MP_i = g(v_T)$

Final exam: 60% of material since midterm
40% of material prior to midterm

- Based on material covered in Lecture.