VCG Mechanisms (Vickrey-Clark-Groves)

Triangle diagram

$$\begin{tikzcd} V \arrow[rr, "F(v)"] \arrow[dr, "a^F(\cdot, \mathcal{I})"'] & & Z \times \mathbb{R}^n \\ & A \arrow[ur, "f(\cdot)"'] & \end{tikzcd}$$

$v \in V = V_1 \times \dots \times V_n$

$\mathcal{I} = (A_1, \dots, A_n; f)$

$f: A_1 \times \dots \times A_n \to Z$

$F(v) = f(a^F(v, \mathcal{I}))$

$F: V_1 \times \dots \times V_n \to Z \times \mathbb{R}^n$

$a^F(\cdot, \mathcal{I}): V_1 \times \dots \times V_n \to A_1 \times \dots \times A_n$

Let $\mathcal{I}$ be an institution. $\mathcal{I}$ is a collection of <u>action sets</u> A_i and <u>rules</u> f.

Z is the outcome space.

Can we design institutions such that equilibrium wrt the institution leads to efficiency? Let $F(v)$ be a mapping from types to efficient allocations. The above question is the central question of mechanism design.

- $V \longrightarrow Z \times \mathbb{R}^n$ direct mechanisms
  - reporting one's type
- $V \longrightarrow A \longrightarrow Z \times \mathbb{R}^n$ indirect mechanisms
  - more realistic

We will be working primarily with direct mechanisms. If such a mechanism exists, then we can (probably) find an indirect mechanism to achieve this result. (ie it is implementable.)

$F: V_1 \times \dots \times V_n \to Z \times \mathbb{R}^n$ is a direct mechanism.

$F_i(v) = (z_i, m_i)$. The distinction between private and public goods can be blurred here. (ie. we can have $F_i(v) = (z, m_i)$)

$$v_i(z) = a z_i - b \frac{z_i^2}{2}$$

- $Z = \{z = (z_i) : \Sigma z_i = 0\}$ private goods
- $Z = \{z = (z_i) : z_i = z \in \mathbb{R}\}$ public goods

QL: $v = (v_1, \dots, v_n)$

$$g(v) = \max\left\{\sum_{i=1}^n v_i(z) : z \in Z\right\}$$

Defn: Let $\pi_i^F(v|v_i'; v_i) = v_i(z_i(v|v_i')) + m_i(v|v_i')$ be the payoff to individual i of type v_i who announces v_i'.

Defn: Define $F_i(v) = (z_i(v), m_i(v))$ to be the allocation assigned under v.

Defn: Let $v = (v_1, \dots, v_n)$. Define the replacement operation by $v|v_i' = (v_1, \dots, v_{i-1}, v_i', v_{i+1}, \dots, v_n)$.

The mechanism computes "an efficient allocation" on the basis of the reports.

The mechanism has no way of inferring the truth from the reports. Can we design a scheme by which each individual rationally chooses to report the truth?

We will have that $\pi_i^F(v|v_i;v_i) = \pi_i^F(v)$

We want $\pi_i^F(v|v_i;v_i) \geq \pi_i^F(v|v_i';v_i)$ $\forall v_i' \in V_i, \forall i, \forall v$.

If this holds, we say that truth-telling is a dominant strategy. Can we find some F such that this holds?

- We rule out the case where v_j enters <u>directly</u> into the payoff of individual i. (This rules out adverse selection and common value problems.)

<u>Defn:</u> a mechanism is <u>budget balancing</u> if $\sum_{i=1}^{n} m_i(v) = 0 \quad \forall v$.

For now, we will <u>not</u> assume budget balancing. Can we find some money payments such that truth-telling is a dominant strategy?

Define $g(v|v_i';v_i)$ as the total gains from trade when i is of type v_i but announces type v_i'.

Define $g(v) \equiv g(v|v_i;v_i)$

Then by definition, we will have $g(v|v_i';v_i) \leq g(v)$ $\forall v_i' \in V_i$.

Suppose $\pi_i^F(v|v_i';v_i) = g(v|v_i';v_i)$ $\forall v_i' \in V_i$. Then we say that individual i fully appropriates.

- Here, $\max_{v_i' \in V_i} \pi_i^F(v|v_i';v_i) = g(v)$

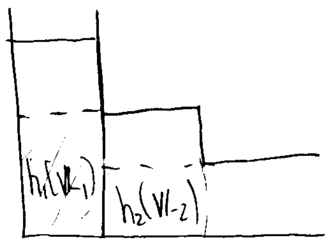
Utility rights

Let $h_i(v_{-i})$ be such that $\pi_i(v|v_i';v_i) = g(v|v_i';v_i) - h_i(v_{-i})$.

We refer to $h_i(v_{-i})$ as the utility rights for the rest of the economy as far as i is concerned.

Then $\max_{v_i' \in V_i} \pi_i^F(v|v_i';v_i) = g(v) - h_i(v_{-i})$

One candidate for $h_i(v_{-i})$ is $h_i(v_{-i}) = g(v_{-i})$



Let p be the price that would be charged if you were not there. (second price sealed-bid auction.)

This is an example of the above reward scheme.

$$\pi_i^F(v\!\!/|v_i'; v_i) = v_i(z_i(v\!\!/|v_i')) + m_i(v\!\!/|v_i') \quad (1)$$

$$= g(v\!\!/|v_i', v_i) - h_i(v\!\!/_{-i})$$

$$= \sum_{j \neq i} v_j(z_j(v\!\!/|v_i')) + v_i(z_i(v\!\!/|v_i')) - h_i(v\!\!/_{-i}) \quad (2)$$

Looking at (1) and (2), we have

$$m_i(v\!\!/|v_i') = \sum_{j \neq i} v_j(z_j(v\!\!/|v_i')) - h_i(v\!\!/_{-i})$$

(compare this to $SOC_{-i}(-z_i) = p(-z_i)$)

If $m_i(v\!\!/|v_i') = \sum_{j \neq i} v_j(z_j(v\!\!/|v_i')) - h_i(v\!\!/_{-i})$, then truth-telling is a dominant strategy.