Public goods
Preference revelation
Truth telling mechanisms

n buyers $i \in \{1,..,n\}$
1 seller $i \in \{0\}$

Public		Private
$z_i - z_0 = 0 \ \forall i$	Quantity Clearing	$\sum_{i=1}^{n} z_i - z_0 = 0$
$\sum_{i=1}^{n} p^i - p^0 = 0$	Price Clearing	$p^i - p_0 = 0 \ \forall i$

$=$ non-excludable (national defense)
$\leq$ excludable (pay TV)

where p^i denotes the price vector faced by individual i.

Defn: a Lindahl equilibrium for the economy $(v_0, v_1, .., v_n)$ is a vector $[(p^i), p^0, (z_i), z_0]$ satisfying

i) $z_i - z_0 \leq 0 \quad \forall i$

ii) $\sum_{i=1}^{n} p^i - p^0 = 0$

iii) $v_i^*(p^i) = v_i(z_i) - p^i z_i \qquad \forall i$

(v) $v_0^*(p^0) = p^0 z_0 \underbrace{- c_0(z_0)}_{v_0(z_0)}$

Question: Is a Lindahl equilibrium efficient?

Defn: an allocation z is efficient if $z = \text{argmax}\{[\sum v_i](z) - c_0(z)\}$

assuming concavity and differentiability, the FOCs are

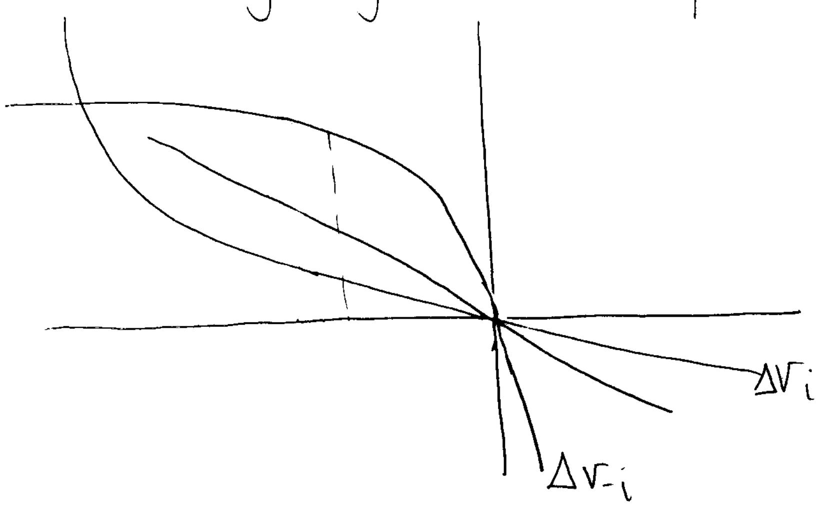
$$\underbrace{[\sum v_i'](z)}_{\text{sum of marginal utilities}} - \underbrace{c_0'(z)}_{\text{marginal cost}} = 0$$

Proposition: a LE is efficient.

This type of pricing is rather naïve. In LE, p^i is determined by one's marginal utility, but why would anyone report his/her marginal utility?

Note the similarities to the notion of joint supply, though. In this respect, it may be somewhat useful. How do we reconcile the differences between these two interpretations?

- asymmetric information in the public goods situation (people do not fully appropriate wrt their announcements.)
- everybody is a monopsonist

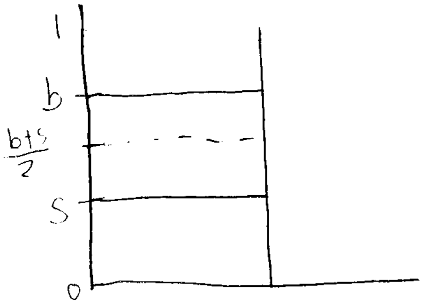


Can individual i misrepresent his/her preferences and be made better off?

Yes.

Truth-telling is not incentive compatible in either the private goods or public goods situation.

- In public goods, these incentives get "worse" with large numbers
- In private goods, they get "better" with large numbers



one buyer, one seller

$B = S = [0,1]$

Trading rule:

$$z(b,s) = \begin{cases} 1 & b \geq s \\ 0 & b < s \end{cases}$$

How much should the buyer pay the seller? Suppose

$$m_B(b,s) = \begin{cases} -\left[\frac{b+s}{2}\right] & b \geq s \\ 0 & b < s \end{cases}$$

Does it pay to manipulate? Yes - the buyer would want to announce some b' s.t. $s \leq b' < b$

Suppose $Prob(b,s) = \lambda(b,s) = \lambda(b) \cdot \lambda(s)$

$$\underbrace{G(b)}_{\text{gains from trade for buyer } b} = \int (b-s)\, z(b,s)\, \underbrace{d\lambda(s)}_{=ds}$$

$$\underbrace{G(b';b)}_{\text{gains to buyer } b \text{ who reports } b'} = \int (b-s)\, z(b',s)\, \underbrace{d\lambda(s)}_{=ds}$$

$$G(s) = \int (b-s)\, z(b,s)\, \underbrace{d\lambda(b)}_{=db}$$

$$\underbrace{G(s';s)}_{\text{gains to buyer } s \text{ who reports } s'} = \int (b-s)\, z(b,s')\, d\lambda(b)$$

What is the optimal strategy $\beta(b)=b^*$ for buyer b?
(ie what should buyer b announce?)
Similarly, what is the optimal strategy $\sigma(s)=s'$ for seller s?

(1) $\underbrace{U_b(\beta(b),\sigma)}$ $= \int b\, z(\beta(b),\sigma(s))\,d\lambda(s) + \int m_B(\beta(b),\sigma(s))\,d\lambda(s)$

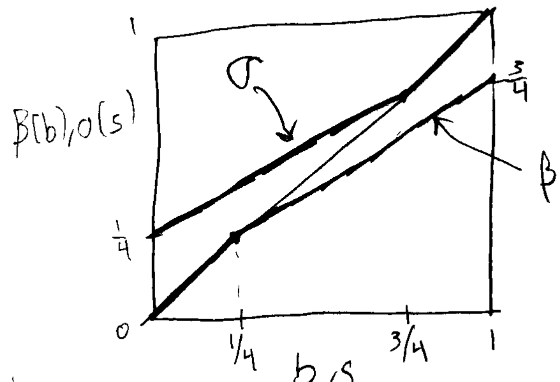
utility to buyer with valuation b and who uses strategy $\beta(b)$ facing a seller using strategy σ.

$\geq \int b\, z(b',\sigma(s))\,d\lambda(s) + \int m_B(b',\sigma(s))\,d\lambda(s) \quad \forall b' \in B$

(2) $U_s(\beta,\sigma(s)) = \int s\, z(\beta(b),\sigma(s))\,d\lambda(b) + \int m_s(\beta(b),\sigma(s))\,d\lambda(s)$

$\geq \int s\, z(\beta(b),s')\,d\lambda(b) + \int m_s(\beta(b),s')\,d\lambda(b) \quad \forall s' \in S$

If (β,σ) satisfies (1) and (2), then we say that (β,σ) is a <u>Bayesian Nash Equilibrium</u>



$\left.\begin{array}{l}\sigma(s)=s\\ \beta(b)=b\end{array}\right\}$ truth-telling

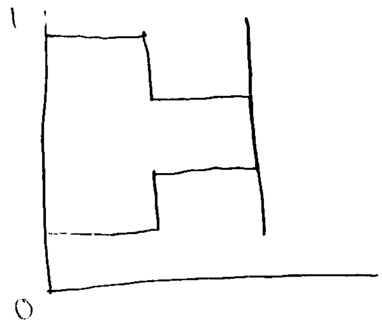
$b' = \min\{\frac{2}{3}b + \frac{1}{12}, b\}$

$s' = \max\{\frac{2}{3}s + \frac{1}{4}, s\}$

It is optimal for buyers with $b > \frac{1}{4}$ to "shade down"

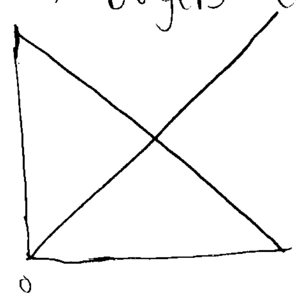
It is optimal for sellers with $s < \frac{3}{4}$ to "shade down."

Next suppose there are two buyers and two sellers
Possible realization



What would the strategy look like?
- It would be closer to the 45° line.

As # buyers and # sellers goes to infinity, we approach



The mechanism cannot be manipulated
- Perfect competition (full appropriation)
- telling the truth becomes the optimal strategy. (weakly optimal)

Suppose the money payment is changed s.t.

$m_b(b,s) = -s$

Then the buyer's optimal strategy is to tell the truth (full appropriation). The seller will always lie upwards.