Efficiency in Economics and Games
FA and NC
Full appropriation and externalities
Real versus pecuniary externalities

Game

$v \in V = V_1 \times \ldots \times V_n$

$\pi_i(v)$

OE: $\pi_i(v) \geq \pi_i(v_{-i}, v_i') \quad \forall v_i' \in V_i \ \forall i$

Economy

$v$

$z_i(v), v_i^*(p(v))$

$\sum_{i=1}^{n} v_i(z_i(v)) = g(v)$
(conditional efficiency)

Efficiency: $v^*$ is efficient if
$v^* = \underset{v \in V}{\operatorname{argmax}}\ g(v)$

Under what conditions does $v^*$ being an OE $\Rightarrow v^*$ is efficient?

In this framework, "people move before prices are set."

Define $\Delta\pi_i(v; v_i') \equiv \pi_i(v_{-i}, v_i') - \pi_i(v)$
$\Delta g(v; v_i') \equiv g(v_{-i}, v_i') - g(v)$

$\Delta\pi_i(v; v_i') \cdot \Delta g(v; v_i') < 0 \Rightarrow$ incentives are not properly aligned. While this may not necessarily preclude efficiency, this is not a good sign.

We want to have $\Delta\pi_i(v; v_i') \cdot \Delta g(v; v_i') \geq 0$.

Defn. We say a game exhibits full appropriation if
$\Delta\pi_i(v; v_i') = \Delta g(v; v_i') \quad \forall v \in V, \forall v_i' \in V_i \ \forall i$

Prop: If a game exhibits full appropriation, and $v$ is an occupational equilibrium, then $\Delta g(v; v_i') \leq 0$ $\forall v_i' \in V_i$ $\forall i$.

"everybody is doing the 'right' thing in the sense that if he/she did anything else, social gains would be nonpositive."

Define $\Delta g(v; v') = g(v') - g(v)$ where $v' = (v_1', \ldots, v_n')$

Defn: $v$ is <u>efficient</u> if $\Delta g(v; v') \leq 0$ $\forall v' \in V$.

Question: Does FA and OE $\Rightarrow$ efficiency? ie if $\Delta g(v; v_i') \leq 0$ $\forall v_i' \in V_i$ $\forall i$, is it necessarily the case that $\Delta g(v; v') \leq 0$ $\forall v' \in V$?

Let $v' = (v_1', \ldots, v_n')$. It may be the case that $\Delta g(v; v_1') + \ldots + \Delta g(v; v_n') < \Delta g(v; v')$. That is, the whole may be bigger than the sum of the parts.

Defn: If $\sum_{i=1}^{n} \Delta g(v; v_i') \geq \Delta g(v; v')$ $\forall v, v' \in V$, then we say the game has <u>no complementarities</u> (NC).

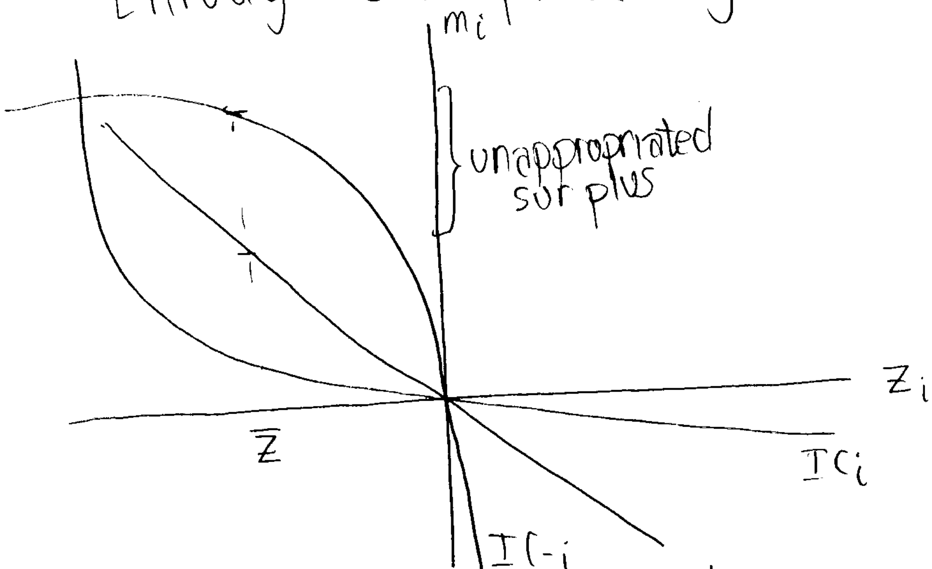
Prop: Suppose a game exhibits full appropriation and no complementarities. Let $v$ be an occupational equilibrium. Then $v$ is efficient.

i.e. $\underbrace{FA + NC}_{\text{the environment}} + \underbrace{OE}_{\text{self-interest}} \Rightarrow$ Efficiency

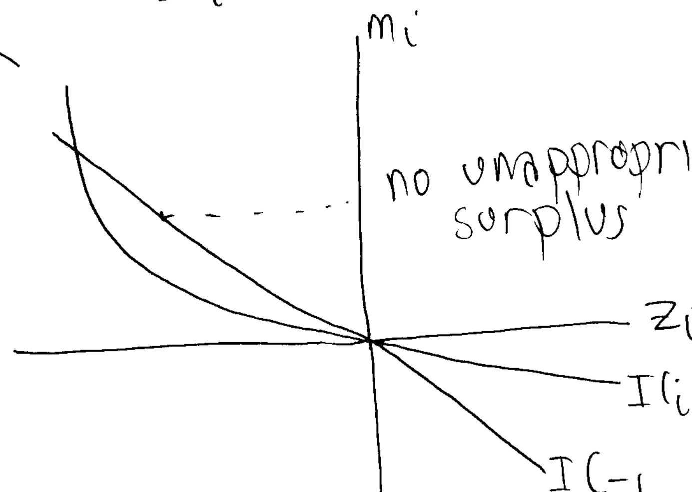
Real versus pecuniary externalities

Defn: a <u>real externality</u> is the direct impact of individual $i$'s actions on individual $j$'s payoff, $j \neq i$, that does not go through the price system.

Defn: a <u>pecuniary externality</u> is the indirect impact of individual $i$'s actions on individual $j$'s payoff, $j \neq i$, through the price system.




This is not the picture of perfectly competitive equilibrium. The individual creates positive externalities.



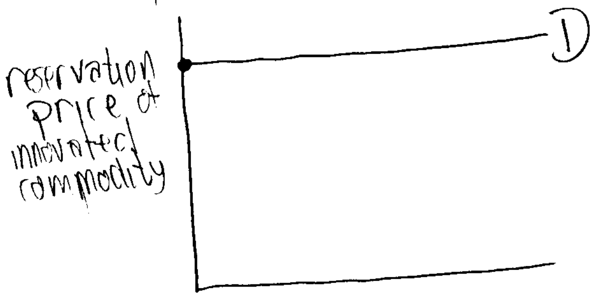

Here, there are no externalities either positive or negative. Perfect competition. Full appropriation

Defn: Let $\Delta p_c(v; v_i') = p_c(v_{-i}; v_i') - p_c(v)$

Defn: a game exhibits <u>perfectly elastic demand and supply</u> (PEDS) if $\Delta p_c(v; v_i') = 0 \ \forall c \in L(v)$ $\forall v \ \forall v_i' \in V_i \ \forall i$. (ie no one can affect prices.)

Suppose we do not have standardized commodities: $L(v_{-i}; v_i') \neq L(v)$. Now we have a modeling problem. What is the price of the new commodities: $[L(v_{-i}; v_i') \setminus L(v)$ assuming we have $L(v) \subset L(v_{-i}; v_i')]$

Suppose you are a perfectly competitive innovator:



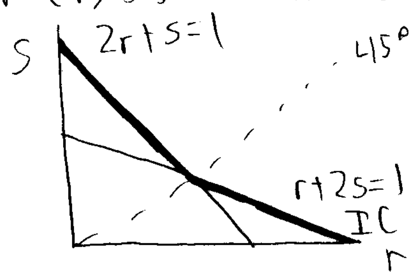

This is based on knowing the economy reservation price of the innovated commodity for your innovated commodity - a strong assumption, but there could be some justification (ie the person who creates the new commodity has a good idea of the market for his/her commodity.)

How do we know that the "right" set of commodities has been created? (we can have inefficient OE when NC is not satisfied)

Suppose there are two commodities that have not been innovated: r - hardware, s - software

Buyers have preferences:

$v(r,s) = \min\{2r+s, r+2s\}$



These utility functions are concave and not differentiable: The directional derivative is superadditive but not additive.

$Dv((0,0);(1,0)) = Dv((0,0);(0,1)) = 1$ but

$Dv((0,0);(1,1)) = 3$

$\Rightarrow Dv((0,0);(1,0)) + Dv((0,0);(0,1)) = 2 < 3 = Dv((0,0);(1,0)+(0,1))$

$\Rightarrow$ superadditivity.

This superadditivity is a problem since individually, we look only at our own part of the problem, but from society's perspective, we should look at both parts of the problem.

With differentiability, we have additivity, so we <u>need</u> only look at our own part of the problem.