- Replicating assignment model
- Competition / Strategic behavior / Incentives
- another version of the First Theorem

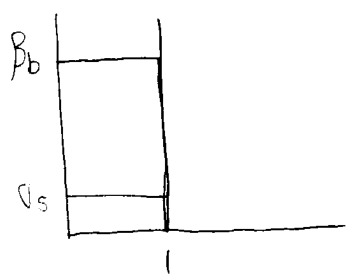
Typically, $\sum_{i=1}^{n} MP_i \geq v_I(0)$ and $MP_i \geq v_i^*(p)$

where p is PTE prices

as we replicate, $MP_i^k \to v_i^*(p)$

ie $MP_i^k \leq MP_i^{k+1} \leq \dots$ and $MP_i^k \downarrow v_i^*(p)$.

In the replica world, the substitution forces outweigh the complementation forces, and MP_i^k is decreasing in k.



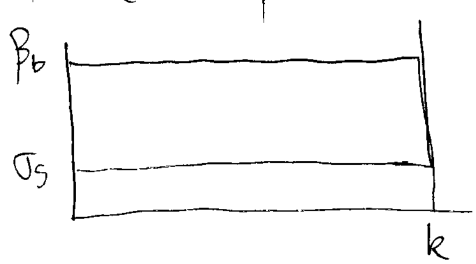
Simple assignment model:

- seller can supply any $z_s \in [-1, 0]$ at cost σ_s
- buyer can consume any $z_b \in [0, 1]$ at benefit β_b

G = Gains from trade = $\beta_b - \sigma_s$

$MP_b = MP_s = G \Rightarrow MP_b + MP_s > G$

If we replicate:

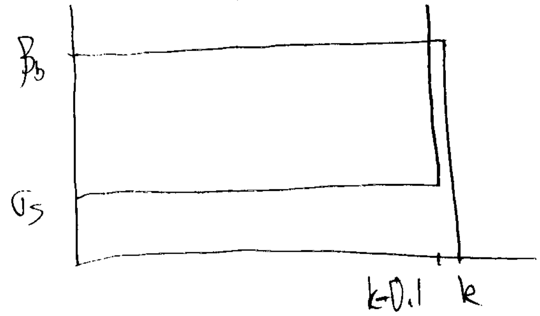


$MP_b = MP_s = G \;\forall\; b, s$

$\Rightarrow \sum MP_b + \sum MP_s = 2kG > kG$

Prices can be any $p \in [\sigma_s, \beta_b]$

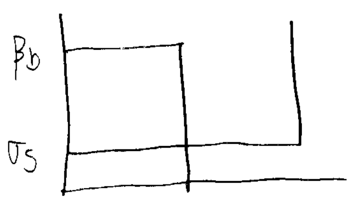
Suppose <u>any</u> seller said "I only want to supply 9/10 of a unit"



Now, $p = \beta_b$
a small change in supply has a large impact on prices.
Buyers can similarly influence prices.

This is an example where, with large numbers monopoly power does not go away. That is, the flattening effect of large numbers does not hold.

This is a knife edge case: Suppose # sellers ≠ # buyers
We have full appropriation.
∘ perfect competition



If we took an assignment model with large numbers of individuals at random, we would most likely have full appropriation.

Intense (perfect) competition is a necessary* condition for self-interested behavior to lead to efficiency.

(*) Qualification: suppose $v(z_1, z_2) = z_2$
$v(z_1, z_2) = z_1$

$Z_1 = \{z : z \geq \begin{bmatrix} -1 \\ 0 \end{bmatrix}\}$ 1 has what 2 wants and
$Z_2 = \{z : z \geq \begin{bmatrix} 0 \\ -1 \end{bmatrix}\}$ 2 has what 1 wants.

In this situation, we do not need competition for efficiency, because there is <u>no</u> conflict.

Discuss the connection:

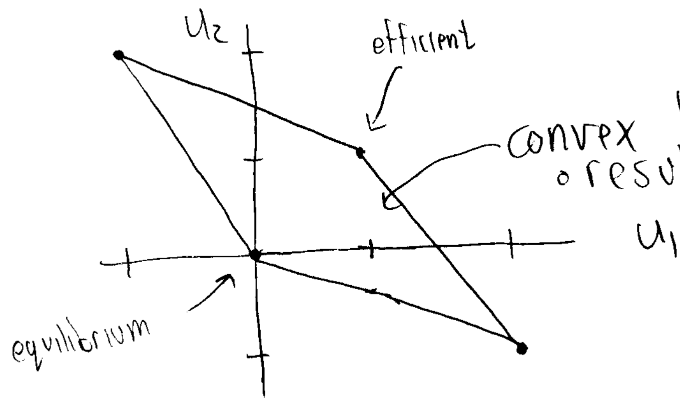
G E	Game Theory
(Price-taking behavior)	(strategic behavior)

In perfect competition, price-taking becomes the best strategy. If this is not the case, gains from trade become dissipated.

I \ II	U	D
U	1, 1	-1, 2
D	2, -1	0, 0

Prisoner's dilemma
- There is a discrepancy between equilibrium and efficiency.

It is almost always the case that the set of NE do not intersect the set of efficient allocations.



First example of strategic behavior

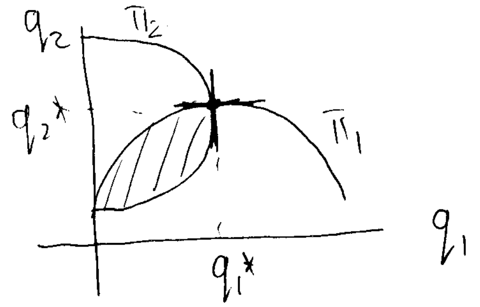
demand: $q = A - p \Rightarrow p = A - q$

$$\Pi_1(q_1, q_2) = (A - (q_1 + q_2))\, q_1 - C_1(q_1)$$

$$\Pi_2(q_1, q_2) = (A - (q_1 + q_2))\, q_2 - C_2(q_2)$$

Equilibrium conditions are: $\frac{\partial \Pi_i}{\partial q_i}(q_1, q_2) = 0 \quad i = 1, 2$

(Cournot equilibrium)



Profits increase going in the SW direction

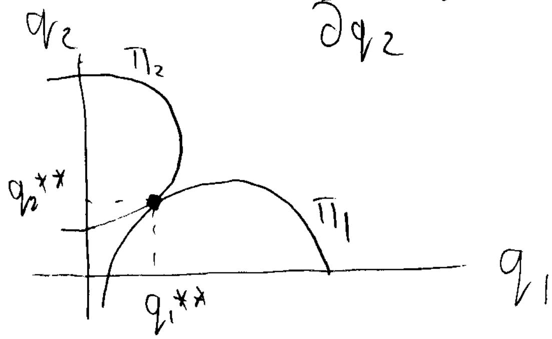
- region of mutual advantage.

Here, equilibrium (q_1^*, q_2^*) is not efficient for these two players. (We ignore the buyers.)

How can the sellers exploit this? ie how can we

$$\max_{q_1, q_2} \pi_1(q_1, q_2) + \pi_2(q_1, q_2)$$

FOCs:
$$\left.\begin{aligned} \frac{\partial \pi_1(q_1, q_2)}{\partial q_1} + \frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = 0 \\ \frac{\partial \pi_1(q_1, q_2)}{\partial q_2} + \frac{\partial \pi_2(q_1, q_2)}{\partial q_2} = 0 \end{aligned}\right\} \text{Cartel solution}$$



What is happening here? There are externalities. q_i affects the profits of q_{-i}. Selfish behavior leads to bad things. (Contradicting Hayek)

Full appropriation - you are not creating any externalities you are not paying for (or getting paid for.)

We will connect GE and game theory using the occupational choice model.

<u>Stage 1:</u> Choose occupation

<u>Stage 2:</u> Everybody maximizes in a price-taking environment

What conditions suffice for the equilibrium of this game to be efficient?

We can summarize a QLGE model by $V = (v_i)_{i \in I}$.

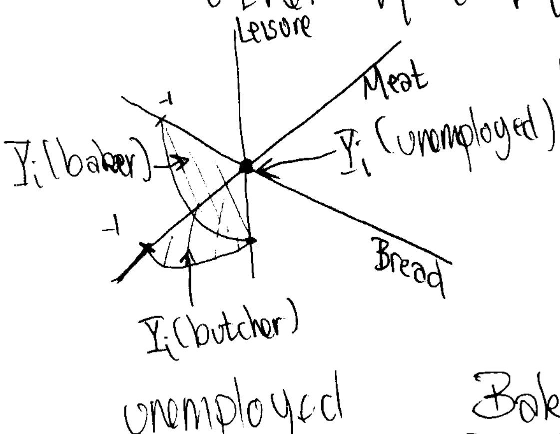
Suppose individual consumers are also producers.

Let V_i be the set of v_i that i can choose.

$v_i \in V_i = \{\text{occupations / strategies}\}$

When you choose v_i, you are implicitly choosing $Y_i(v_i)$, your set of production possibilities.

- Choosing v_i precludes you from choosing any other $v_i' \in V_i$



ie. can choose to be a baker or a butcher or be unemployed. $Y_i(\text{unemployed}) = 0$

$$
\begin{array}{cccc}
 & \text{unemployed} & \text{Baker} & \text{Bread} \\
\begin{matrix}\text{Leisure}\\ \text{Bread}\\ \text{Meat}\end{matrix} & \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} & \begin{bmatrix} 1/2 \\ -1 \\ 0 \end{bmatrix} & \begin{bmatrix} 1/2 \\ 0 \\ -1 \end{bmatrix}
\end{array}
$$

Suppose overall utility function for i looks like

$$\max_{y_i \in Y_i(v_i) \cap \mathbb{R}^{l(v)}} U_i(\omega_i + y_i + z_i \mid v_i) = v_i(z_i)$$

(ω_i: initial endowment)

$v = (v_i)_{i \in I}$ is like a strategy n-tuple. (ie each person has chosen an occupation) where $I = \{1, \dots, n\}$

Suppose l is the set of all commodities. This is a huge set. Let $l(v) = \{c : \exists i$ st. $z_{ic} < 0$ that is feasible for $i\}$ where $c = 1, \dots, l$.

(ie $l(v)$ is the set of commodities that can be produced under v.)

Here, the choice of commodities is a part of the equilibrium problem.

$$v \in V = V_1 \times \dots \times V_n$$

Given a particular v, we can easily find a price-taking equilibrium.

$$v \longmapsto \text{PTE} \quad p(v) \in \mathbb{R}^{l(v)}$$

The payoff to i is $v_i^*(p(v))$

where $v_i^*(p(v)) = \max\{v_i(z_i) - p(v) \cdot z_i : z_i \in \mathbb{R}^{l(v)}\}$

Thus,

$$v \longrightarrow p(v) \longrightarrow v_i^*(p(v)) \longrightarrow \pi_i(v)$$

Everything becomes a function of v.

Therefore, we now have a game in normal form.
What is an equilibrium in this game?

Defn: an **occupational equilibrium** is an occupational choice $\mathcal{V}$ s.t. $\pi_i(\mathcal{V}) \geq \pi_i(\mathcal{V}_{-i}, v_i') \; \forall v_i' \in V_i \; \forall i$.
(i.e. $\mathcal{V}$ is a Nash equilibrium.)

The fact that in this model, not all commodities are produced (ie $\ell \neq \ell(\mathcal{V})$) allows for innovation.

Under what conditions is an occupational equilibrium efficient?

Let $g(\mathcal{V}) = \max_{(z_i)} \left\{ \sum_{i \in I} v_i(z_i) : z_i \in \mathbb{R}^{\ell(\mathcal{V})}, \sum_{i \in I} z_i = 0 \right\}$

We will necessarily have $\sum_{i \in I} \pi_i(\mathcal{V}) = g(\mathcal{V})$ by first welfare theorem. But this is not the definition of efficiency in light of occupational choice.

An occupational choice $\mathcal{V}$ is **Pareto optimal** if $\mathcal{V} = \arg\max_{\mathcal{V}' \in V} g(\mathcal{V}')$ where $V = V_1 \times \ldots \times V_n$.

In general, we will have that when $\mathcal{V}$ is an occupational equilibrium, $\mathcal{V}$ is not Pareto optimal.