Commodity / Individual Margin
Full appropriation
Integral Optimality
Flattening / Convexifying Effects

$MP_i = v_I(0) - v_{-i}(0)$

We have illustrated that $\sum_{i=1}^{n} MP_i \geq v_I(0)$ with strict inequality often.

PTE for $\{(v_i)\}$ is a $[(z_i), p]$ s.t.

i) $\Sigma z_i = 0$, $m_i = -pz_i \; \forall i$

ii) $v_i^*(p) = v_i(z_i) - pz_i \quad \forall i$

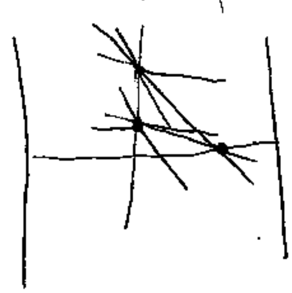
$$\Rightarrow \sum_{i=1}^{n} v_i^*(p) = \sum_{i=1}^{n}[v_i(z_i) - pz_i] = \sum_{i=1}^{n} v_i(z_i) - p\underbrace{\sum_{i=1}^{n} z_i}_{=0} = \sum_{i=1}^{n} v_i(z_i)$$

We know that PTE are PO $\Rightarrow \sum_{i=1}^{n} v_i(z_i) = v_I(0)$

Since $v_I(0)$ is the total pie, $\sum_{i=1}^{n} v_i(z_i) = v_I(0)$ is a way of splitting up the pie.

Under what condition is this division of the pie unique? answer: when the prices are unique

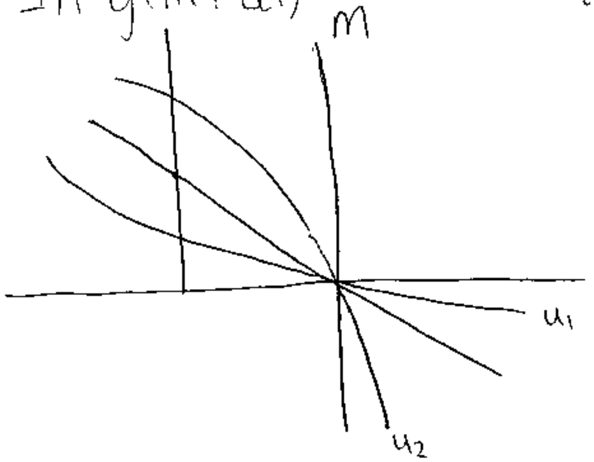
Eg when prices are not unique:



We need only assume differentiability wrt one individual in order for the equilibrium allocation to be unique.

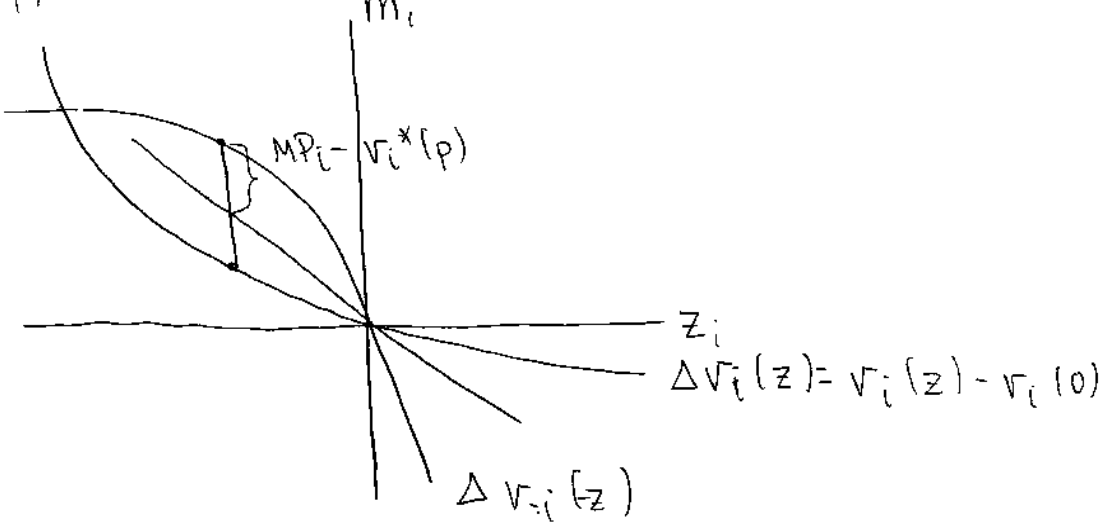
What is the relationship between $\underbrace{v_i^*(p)}_{\text{reward}}$ and $\underbrace{MP_i}_{\text{contribution}}$?

In general, $MP_i \geq v_i^*(p)\ \forall i$.



$$MP_i = \max\{v_i(z) + \underbrace{\Delta v_{-i}(-z)}_{v_{-i}(-z) - v_{-i}(0)}\}$$

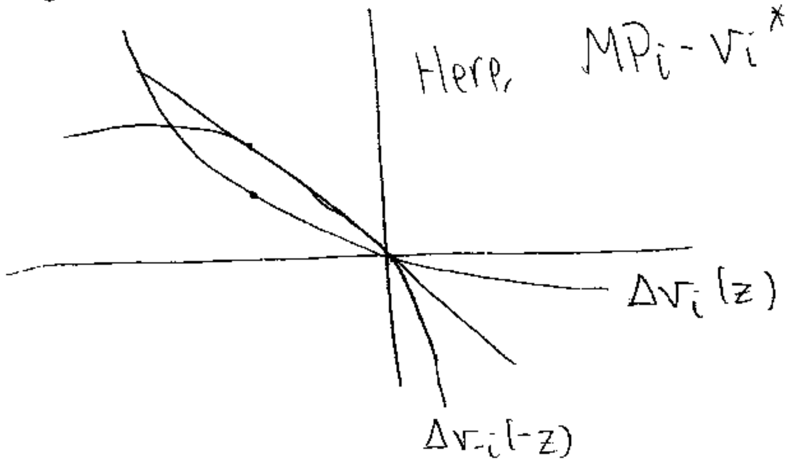
Suppose i is trading with $-i$.



In a perfectly competitive environment, we will have that $MP_i = v_i^*(p)$

This would look like:

Here, $MP_i - v_i^*(p) = 0 \Rightarrow MP_i = v_i^*(p)$

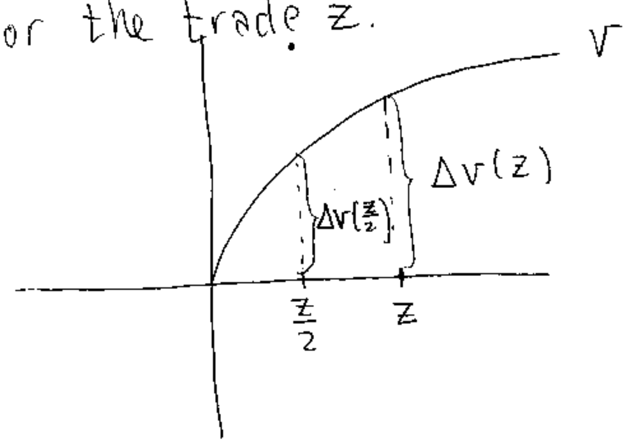


Flattening Effect of Large Numbers

v is concave

$\Delta v(z) = v(z) - v(0)$

$\underbrace{\Delta v_2(z)}_{\text{maximum amount of money we can extract from 2 people for the trade } z.} = \max\{\Delta v(z_1) + \Delta v(z_2) \,;\, z_1 + z_2 = z\}$



assume $v(0) = 0$

$\Delta v_2(z) = 2\,\Delta v\left(\frac{z}{2}\right)$

$\Rightarrow \Delta v_k(z) = k\,\Delta v\left(\frac{z}{k}\right)$

What is $\lim_{k\to\infty} \Delta v_k(z)$?

$$\lim_{k\to\infty} \Delta v_k(z) = \lim_{k\to\infty} k\, \Delta v\left(\frac{z}{k}\right)$$

$$= \lim_{k\to\infty} \frac{v\left(\frac{1}{k} z\right) - v(0)}{\frac{1}{k}}$$

$$= Dv(0; z)$$

$$\Delta v_I(z) = v_I(z) - v_I(0)$$

assume v_i concave $\forall i$

$$\Delta v_{kI}(z) = k\, \Delta v_I\left(\frac{z}{k}\right)$$

$$\Rightarrow \lim_{k\to\infty} \Delta v_{kI}(z) = \lim_{k\to\infty} \frac{v_I\left(\frac{1}{k} z\right) - v_I(0)}{\frac{1}{k}}$$

$$= D v_I(0; z)$$

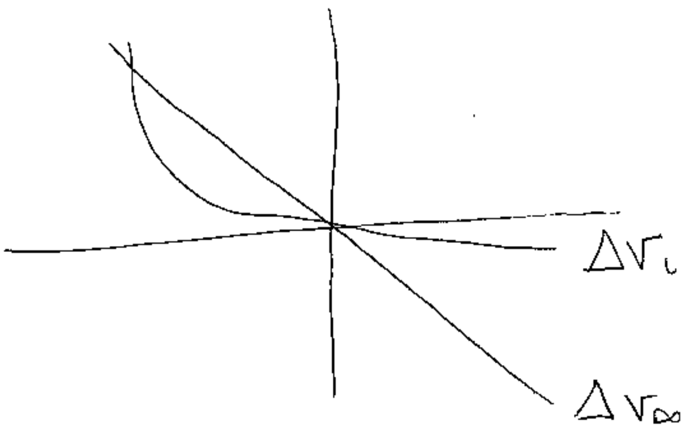
"This quantity z is very small relative to the scale of the economy.

Let $\mathcal{E} = [(v_i)]$ be an economy
Let $\mathcal{E}^k = \{\mathcal{E}_1, \ldots, \mathcal{E}_k\}$ where $\mathcal{E}_j = \mathcal{E}$ $\forall j$ be the k-replica of $\mathcal{E}$

From $\mathcal{E}$, we can derive v_I. From $\mathcal{E}^k$, we can derive v_{kI}

Here, $v_{-i} = v_{kI \setminus \{i\}}$

We will pretend that $\Delta v_{kI\setminus\{i\}} - \Delta v_{kI} \to 0$ as $k \to \infty$. If we have this assumption, then



where $\Delta v_\infty = \lim_{k\to\infty} \Delta v_{kI} = \lim_{k\to\infty} \Delta v_{kI\setminus\{i\}}$

Differentiability gives us determinacy

Suppose v is concave
$\Rightarrow Dv(0;\cdot)$ is positively homogeneous and super-additive.

Production
$v_{pro}(z) = -c(z)$ where $c(z)$ is the money input to produce z.

another possibility is that $c(z) = \begin{cases} 0 & \forall z \in Y \subseteq \mathbb{R}^\ell \\ \infty & \forall z \notin Y \end{cases}$

i.e. $c(z)$ is an indicator function for the production set.

$V_{consumer}(z) = Bz - Az^2$ where z is a scalar

$\frac{\partial V_{con}(z)}{\partial z} = B - 2Az$ (marginal utility)

The FOC is $B - 2Az = p$ (linear demand curve)

$V_{producer}(z) = -[c(z)] = -[Dz^2]$

$\frac{\partial c(z)}{\partial z} = 2Dz$ (marginal cost)

The FOC is $2Dz = p$

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assignment model

2 groups: buyers and sellers

$b \in B = \{1, \ldots, n\}$ $s \in S = \{1, \ldots, m\}$

If b and s get together, there is a "value" for the match: $V(b,s)$. Each b can be matched with exactly one s and conversely.

The data of an assignment model is B, S, and the matrix of values V

$$V = \begin{array}{cc} & S \\ B & \begin{bmatrix} & & \\ & V(b,s) & \\ & & \end{bmatrix} \end{array}$$

The economic problem is how optimally to make assignments.

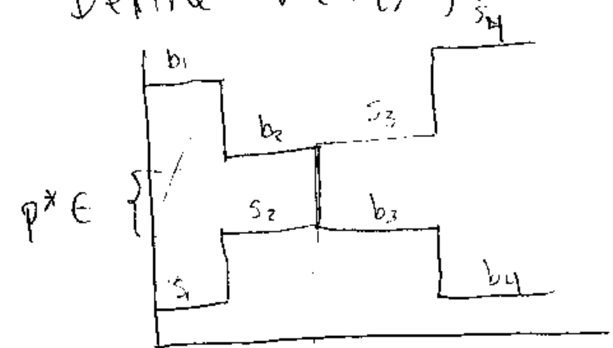
- The dual is: given certain matches, how is the surplus split up?

Special case:

Suppose the buyers B are buyers of a homogeneous commodity where each buyer is characterized completely by her reservation values: $b_1, \ldots, b_n$

The sellers S are completely characterized by reservation values (or cost of production): $s_1, \ldots, s_m$.

Define $V(b_i, s_j) = \max\{b_i - s_j, 0\}$

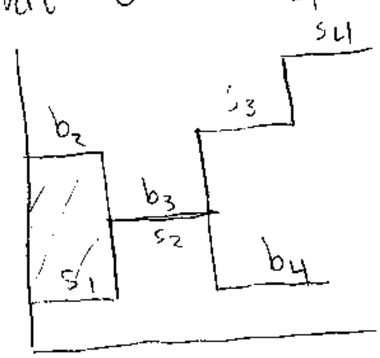


The efficient allocation is q^* supported by any prices $p^* \in [s_2, b_2]$

q^* = ?

$V_I(0) = (b_1 - s_1) + (b_2 - s_2)$

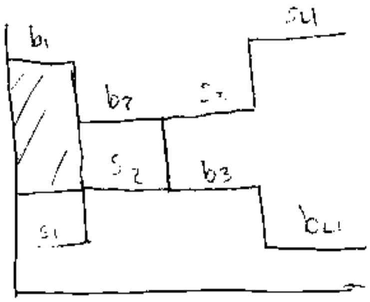
What is MP_{b_1}? Pull b_1 out of the market



$V_{-b_1}(0) = (b_2 - s_1)$

$MP_{b_1} = V_I(0) - V_{-b_1}(0) = (b_1 - s_1) + (b_2 - s_2) - (b_2 - s_1)$

$= b_1 - s_2$



The shaded area is MP_{b_1}

Is there a price at which b_1 will get her marginal product?

- Yes. Choose $p = s_2$.