David Rahman - a model of organized competition
What is the Shapley value?
Jeami allocation of commodities, tasks, payoffs?

$v_t(a_t, z_t) = \sum_{i \in t \cup \{0\}} v_t^i(a_t, z_t)$
$a_t \in A^t$, $z_t \in \mathbb{R}^\ell$

personalized wages in a Lindahl equilibrium.

Review Session

Public Good:
$\{0,1\}$ $\quad \mathbb{v} = (v_i) \in V = V_1 \times \dots \times V_n$

a) $\mathbb{1}_i(\mathbb{v}) = \begin{cases} 1 & \text{if } \sum v_i > 0 \\ 0 & \text{else} \end{cases}$ is efficient

b) $m_i(\mathbb{v}) = 0 \;\forall i$ does not work
Suppose $\sum v_i > 0$, $v_k < 0$ for some k. $\Rightarrow$ No incentive for $\hat{v}_k \neq -\infty$

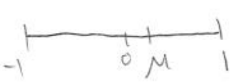
c) $h_i(\mathbb{v}_{-i})$: utility rights of $-i$
$Ext_i = \left(\sum_{j \neq i} v_j\right) \mathbb{1}_I(\mathbb{v}) - h_i(\mathbb{v}_{-i})$ $\quad$ i's external effects.

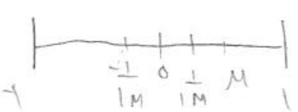
$m_i(\mathbb{v}|v_i') = \sum_{j \neq i} v_j(z(\mathbb{v}|v_i')) - h_i(\mathbb{v}_{-i})$

Let $h_i(\mathbb{v}_{-i}) = g_i($
$u = v_i(z(\mathbb{v})) + m_i(\mathbb{v}) = g(\mathbb{v}) - h_i(\mathbb{v}_{-i}) \stackrel{\downarrow}{=} g(\mathbb{v}) - g_i(\mathbb{v}_{-i})$

d) Show that $m_i(\mathbb{v}) \leq 0$
Suppose at first they did not have the statue.
See what happens if pivotal / not pivotal, etc.

e) Let $V_i = [-1, 1]$. Suppose $\mu > 0$





Outside $\left[-\frac{1}{1M}, \frac{1}{1M}\right]$, no one is pivotal.

$\left|\hat{\mu} = \frac{1}{N}\sum v_i\right| \geq \frac{1}{N} \Rightarrow \left|\sum v_i\right| \geq 1 \Rightarrow$ Cannot manipulate answer with one person.

$\underbrace{\Pr(\text{Pivotal}) \left|\sum_{j \neq i} v_i\right|}_{m_i\,?}$ $\quad$ as $N \to \infty$, $\Pr(\text{pivotal}) \downarrow$ and $\left|\sum_{j \neq i} v_i\right| \uparrow$

But $\Pr(\text{pivotal})\downarrow$ stronger than $\left|\sum_{j \neq i} v_i\right|\uparrow$

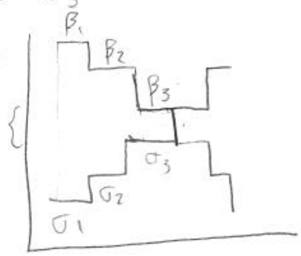
f) $\forall n \ \exists \mathbb{v} \ni m_i^{FA}(\mathbb{v}) < 0$ $\{-1, 1, -1, -\} \Rightarrow 0$

g) ??

2. Private good

$v_b = \beta_b$ $\mathbb{v} = (v_{b_j}, v_{s_k}) \in V = \times_j V_{b_j} \times_k V_{s_k}$

$v_s = \sigma_s$

a) 

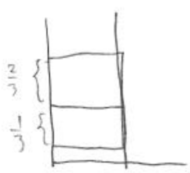
$EF: M \equiv \beta_M \geq \sigma_M, \ \beta_{M+1} < \sigma_{M+1}$

in this case, $M = 3$

$$z_{b_k} = \begin{cases} 1 & k \leq M \\ 0 & \text{else} \end{cases}$$

$$z_{s_k} = \begin{cases} -1 & k \leq M \\ 0 & \text{else} \end{cases}$$

b) $m_i(\mathbb{v}) = -p(\mathbb{v}) z_i(\mathbb{v})$. Why doesn't this elicit truthful revelation?

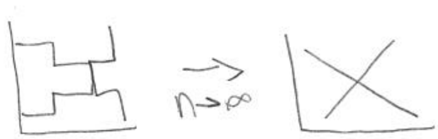


c) $m_i(\mathbb{v}) \equiv \sum_{j \neq i} v_j(z(\mathbb{v})) - h_i(\mathbb{v}_{-i})$

$v_i(z(\mathbb{v})) + m_i(\mathbb{v}) = MP_i$ if $h_i(\mathbb{v}_{-i}) = g(\mathbb{v}_{-i})$

$m_i(\mathbb{v}) \geq -p(\mathbb{v}) z(\mathbb{v})$

e) $V_i = [0, 1]$



Flattening effect

3. Not much difference.