Mechanism Design ↔ Order without design

- "Slightly" new question but not new answer

Blur distinction between public vs private goods and real vs. pecuniary externalities.

Can "social engineers" create superior institutions? Cf. Order without design

└ Hayek's quotes

Mechanism design is the opposite of "order without design"

└ But both work when there is perfect competition.

$V = V_1 \times \dots \times V_n \qquad \mathbf{v} \in V$ vector of characteristics

Want to design an optimal scheme with good incentives.

Let $F: V \longrightarrow$ "physical outcomes" be a mechanism

$F(\mathbf{v}) = (z(\mathbf{v}), m(\mathbf{v}))$ where $m(\mathbf{v}) = (m_1(\mathbf{v}), \dots, m_n(\mathbf{v}))$

What is $z(\mathbf{v})$?

public goods
- Consider the case of costless public goods with $z \in \{0,1\}$
- Suppose $v_i(0) \equiv 0$, associate $v_i(1) \sim v_i$. You are characterized by your utility from $z=1$.

private goods
- another possibility: $z(\mathbf{v}) = (z_1(\mathbf{v}), \dots, z_n(\mathbf{v}))$ where $z_i(\mathbf{v}) \in \mathbb{R}^\ell$ and $\sum z_i(\mathbf{v}) = \omega$
- $v_i(z_i(\mathbf{v}))$

Efficiency: $\sum_{i=1}^{n} v_i(z(\mathbf{v})) = \max \left\{ \sum_{i=1}^{n} v_i(z'(\mathbf{v})) \text{ over all feasible } z'(\mathbf{v}) \right\}$

└ This is necessary

We will not insist on $\sum_{i=1}^{n} m_i(\mathbf{v}) = 0$

Consider the public goods example:

$(\sum v_i) > 0 \Rightarrow z(\mathbf{v}) = 1$

$(\sum v_i) \leq 0 \Rightarrow z(\mathbf{v}) = 0$

Consider the private goods example: Usual definition of efficiency

Efficiency criterion says that choice of $z(\mathbf{v})$ is out of the control of the mechanism designer.

$u_i(z, m_i; v_i) = v_i(z) + m_i$

$\pi_i(\mathbf{v} | v_i; v_i) = u_i(z(\mathbf{v}), m_i(\mathbf{v}); v_i)$

aggregate report, my report, true preferences

$= v_i(z(\mathbf{v})) + m_i(\mathbf{v})$

$\pi_i(\underbrace{\mathbf{v}_{-i} | v_i'}_{\text{aggregate report with my stated preferences}}; v_i) = u_i(z(\mathbf{v}_{-i}, v_i'); m_i(\mathbf{v}_{-i}, v_i'); v_i)$

$= v_i(z(\mathbf{v}_{-i}, v_i')) + m_i(\mathbf{v}_{-i}, v_i')$

What do we want?

$\pi_i(\mathbf{v}|v_i;v_i) \geq \pi_i(\mathbf{v}|v_i';v_i) \quad \forall i \quad \forall v_i' \, \forall \mathbf{v}$ (Incentive Compatibility Constraints)

"replacement operation"

L This constrains truth-telling to being a dominant strategy. (Stronger condition than Nash Equilibrium)

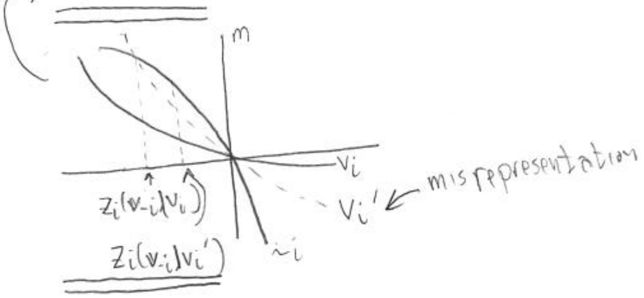
$g(\mathbf{v}) = \sum_{i=1}^{n} v_i(z(\mathbf{v}))$ total gains from trade from perspective of i

$g(\mathbf{v}|v_i';v_i) = \sum_{j \neq i} v_j(z(\mathbf{v}|v_i')) + v_i(z(\mathbf{v}|v_i'))$

Rewriting (in a more confusing way): $g(\mathbf{v}) = g(\mathbf{v}|v_i;v_i)$

Theorem: $g(\mathbf{v}|v_i;v_i) \geq g(\mathbf{v}|v_i';v_i)$

Pf: $z(\mathbf{v})$ was chosen to be the best it could possibly be.



This graph depicts the above statement

Suppose $\pi_i(\mathbf{v}|v_i';v_i) = g(\mathbf{v}|v_i';v_i)$ (ie. you capture all the rewards)

Then $\max_{v_i' \in V_i} \pi_i(\mathbf{v}|v_i';v_i) = \pi_i(\mathbf{v}|v_i;v_i)$ (it pays to tell the truth)

We would need a lot of money to pay everyone the total gains.

Suppose $\pi_i(\mathbf{v}|v_i';v_i) = g(\mathbf{v}|v_i';v_i) - g(\mathbf{v}_{-i})$ or $(= g(\mathbf{v}|v_i';v_i) - h_i(\mathbf{v}_{-i}))$ more general reward scheme

Then $\max_{v_i' \in V_i} \pi_i(\mathbf{v}|v_i';v_i) = \pi_i(\mathbf{v}|v_i;v_i)$

L "Lump sum" does not affect marginal incentives.

We are currently ignoring individual rationality.

What if we require $\sum_{i=1}^{n} m_i = 0$?

$$\frac{\sum MP_i(\mathbf{v}) - g(\mathbf{v})}{n} \longrightarrow 0$$ as we replicate the economy.

L In this situation, it is possible to design such a reward scheme

It is impossible otherwise

Public Goods	Private Goods
$z(\mathbf{v})$	$p(\mathbf{v})$
$p_i(\mathbf{v})$	$z_i(\mathbf{v})$
	PEDS $\Rightarrow$ perfect competition

If your report does not affect z, then this implies perfect competition
L This is a weak result since there is no incentive to tell the truth.

Break

What if $\pi_i(\mathbf{v}|v_i';v_i) = \alpha_i g(\mathbf{v}|v_i';v_i)$ $\alpha_i > 0$, $\sum_{i=1}^{n} \alpha_i = 1$?
L This remark is addressed on pgs 10-11 in the notes.

Returning to $\pi_i(\mathbf{v}|v_i';v_i) = g(\mathbf{v}|v_i';v_i) - h_i(\mathbf{v}_{-i})^{(1)}$ scheme

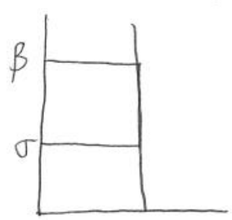
$$\pi_i(\mathbf{v}|v_i';v_i) = v_i(z(\mathbf{v}|v_i')) + m_i(\mathbf{v}|v_i')$$

$$\overset{(1)}{=} \underbrace{\sum_{j\neq i} v_j(z(\mathbf{v}|v_i')) + v_i(z(\mathbf{v}|v_i'))}_{g(\mathbf{v}|v_i';v_i)} - h_i(\mathbf{v}_{-i})$$

$$\Rightarrow m_i(\mathbf{v}|v_i') = \sum_{j\neq i} v_j(z(\mathbf{v}|v_i')) - \underbrace{h_i(\mathbf{v}_{-i})}_{\text{utility rights that must be respected}}$$

↳ i is forced not to create externalities relative to the utility rights.

Will property rights ensure $\sum m_i = 0$?



$$(\beta,\sigma) \in \underbrace{[0,1]}_{V_B} \times \underbrace{[0,1]}_{V_s}$$

$$F(\beta,\sigma) = \begin{cases} 1 & \text{if } \beta \geq \sigma \\ 0 & \text{if } \beta < \sigma \end{cases}$$ where 1 denotes the transaction having occurred.

$$m_b(\beta) + m_s(\sigma) = 0$$

Suppose $\beta, \sigma \sim$ uniform $[0,1]$ and this is common knowledge

$m_b(\beta) = -\frac{\beta-\sigma}{2}$
$m_s(s) = \frac{\beta-\sigma}{2}$
"split the difference rule"
payoff depends on your report

$b(\beta) = \frac{2}{3}\beta + \frac{1}{12} \Rightarrow b(\beta) < \beta$ if $\beta > \frac{1}{4}$

$s(\sigma) = \frac{2}{3}\sigma + \frac{1}{4} \Rightarrow s(\sigma) > \sigma$ if $\sigma < \frac{3}{4}$

Trade occurs if $b(\beta) - s(\sigma) \geq 0 \Rightarrow \beta \geq \sigma + \frac{1}{4} \Rightarrow$ This is not efficient
Chatterjee-Samuelson

Myerson-Satterthwaite – no mechanism is possible.