Pricing in Games
- IC constrained efficiency

IC constrained efficient ≡ second best

Public Goods again.

Mechanism Design

	b_1	b_2
a_1	5,1	0,0
a_2	4,4	1,5

11	12
21	22

$V_1 = (5,0,4,1)$ $X_1 = (x_{11}^1, x_{12}^1, x_{21}^1, x_{22}^1)$

$V_2 = (1,0,4,5)$ $X_2 = (x_{11}^2, x_{12}^2, x_{21}^2, x_{22}^2)$

$u_1 = \begin{cases} V_1 \cdot X_1 & \text{if } X_1 \text{ is a probability distribution} \equiv \Delta \\ -\infty & x_1 \notin \Delta \end{cases}$

$u_2 = \begin{cases} V_2 \cdot X_2 & \text{if } x_2 \in \Delta \\ -\infty & x_2 \notin \Delta \end{cases}$

$V_1[a_1] = (5,0,5,0)$

deviation to always choosing a_1

$V_1[a_2] = (4,1,4,1)$

$V_2[b_1] = (1,1,4,4)$

$V_2[b_2] = (0,0,5,5)$

(1) $V_1[a_1] - V_1 = (0,0,1,-1)$

(2) $V_1[a_2] - V_1 = (-1,1,0,0)$

(3) $V_2[b_1] - V_2 = (0,1,0,-1)$

(4) $V_2[b_2] - V_2 = (-1,0,1,0)$

$u_0 = V_0 \cdot x_0$ $x_0 = (x_{11}^0, x_{12}^0, x_{21}^0, x_{22}^0)$ $x_0 \geq 0$

$V_0 \cdot x_0 = \begin{cases} 0 & \text{if } x_0 \cdot (k) \leq 0 \\ -\infty & \text{else} \end{cases}$ $(k) \in \{(1),(2),(3),(4)\}$

This is where the constraint comes in

Eight prices: $p^1 = (p_{11}^1, p_{12}^1, p_{21}^1, p_{22}^1)$

$p^2 = (p_{11}^2, p_{12}^2, p_{21}^2, p_{22}^2)$

$v_1^* (p^1) \leftarrow x_1$
$v_2^* (p^2) \leftarrow x_2$
$v_0^* (-(p^1 + p^2)) \leftarrow x_0$

In equilibrium, $x_1 = x_2 = x_0$

Total gains:

$\mathbb{v} = (v_1, \ldots, v_n)$, concave

$g(\mathbb{v}) = \sup \{ (\sum_{i=1}^{n} v_i)(z) + v_0(z) \}$

$v_0(z) = -c_0(z)$

1) costless: (a) $v_0(z) = \begin{cases} 0 & \text{if } z \in [0,1] \\ -\infty & \text{else} \end{cases}$ "paint the room a shade of green"

(b) $v_0(z) = \begin{cases} 0 & \text{if } z \in \{0,1\} \\ -\infty & \text{else} \end{cases}$ "paint the room either blue or gray"

2) costly: $c_0(z)$ is increasing convex function

$MP_i(\mathbb{v}) = g(\mathbb{v}) - g(\mathbb{v}_{-i})$

$\sum MP_i(\mathbb{v}) \geq g(\mathbb{v})$ in private goods model – people are complementary

$\sum MP_i(\mathbb{v}) \leq g(\mathbb{v})$ in public goods (costless) model - not complementary

$\sum MP_i(\mathbb{v}) = g(\mathbb{v})$ if z does not change when i is in and i is out

↳ duality relationship with PEDS.

Costly case: Examples: (I) $v(z) = az - \frac{1}{2}z^2$, $c_0(z) = z$

(II) $v(z) = 2z^{1/2}$, $c_0(z) = z$

$g(\mathbb{v}) \overset{?}{\underset{?}{\lesseqgtr}} \sum MP_i(\mathbb{v})$

$2g(\mathbb{v}) \lesseqgtr g(2\mathbb{v}) = \sup \{ [2 \sum_i v_i](z) + v_0(z) \}$

$(<)$

Costly public goods (under "good" conditions) are like increasing returns to scale, Decreasing average cost.

This gives us $\sum MP_i(\mathbb{v}) \geq g(\mathbb{v})$ since people are <u>really</u> complementary

In case (II), as $n \to \infty$, we can reach nirvana

Local public good: a good shared by a subset of the population
- allows for crowding effects (theater example)
  - Competition can arise

Pure public goods - overwhelming complementarities.

Non-manipulable mechanism notes posted on web.

Heuristic Sketch

$MP_i(v) > \pi_i(v) \Rightarrow i$ creates positive externalities

$MP_i(v) < \pi_i(v) \Rightarrow i$ creates negative externalities

$MP_i(v) = \pi_i(v) \Rightarrow$ no externalities
- we can talk about this in mechanism design without respect to competition
- Full appropriation is a good reward scheme.

In occupational choice model,

$v \in V_1 \times \ldots \times V_n$

Now, we will interpret v as a vector of tastes

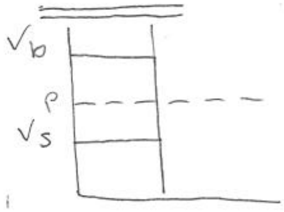
v_i (true tastes) $v_i' \in V_i$ (reported tastes)

How do we get people to report their tastes truthfully?

Weak form of incentive compatibility: no matter what you say, the mechanism will do the same thing.

$v \longrightarrow F(v)$ could be: $F_i(v) = z_i(v)$ ← private good allocation

non-money commodity allocation $F_i(v) = z_i(v) = z(v)$ ← public good allocation.



$v_b \in [0,1]$

$v_s \in [0,1]$

Benevolent s.p. $\Rightarrow$ Want to make sure a trade occurs whenever $v_b \geq v_s$

$p \in (v_s, v_b) \Rightarrow$ Both create externalities

$p = v_s \Rightarrow$ Buyer fully appropriates

$p = v_b \Rightarrow$ Seller fully appropriates

We will drop the requirement that $\sum m_i(v) \neq 0$.