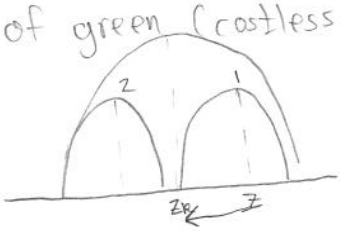
Private goods: $MP_i \geq \pi_i$
Public goods: $MP_i \leq \pi_i$ (if costless)

shade of green (costless production)




$MP_2 = v_1(z_k) + v_2(z_k) - v_1(\bar{z})$

$\pi_2 = v_2(z_k)$

$MP_2 - \pi_2 = v_1(z_k) - v_1(\bar{z}) < 0$

since $v_1(\bar{z}) > v_1(z_k)$

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2004 HW 6 Q3

Grove Mechanism $\quad \mathbb{V} = (v_1, \ldots, v_n)$

$w_i$ – report

A Statue

Basement / Display

Decision $\begin{cases} \text{Display if } \sum w_i \geq 0 \\ \text{Basement else} \end{cases}$

a) Provide a reward scheme that is IC.

$\mathbb{1}_{\{\sum w_i \geq 0\}} = \begin{cases} 1 & \text{if } \sum w_i \geq 0 \\ 0 & \text{else} \end{cases}$

$MP_i = \mathbb{1}_{\{\sum_{j=1}^n v_j \geq 0\}} \cdot \sum_{j=1}^n v_j - \mathbb{1}_{\{\sum_{j \neq i} v_j \geq 0\}} \sum_{j \neq i} v_j$

example: $\sum_{j \neq i} v_j \geq 0, \ \sum_{j=1}^n v_j < 0 \Rightarrow MP_i < 0$

$MP_i \Leftarrow v_i \cdot \left(\mathbb{1}_{\{w_i + \sum_{j \neq i} v_j \geq 0\}}\right) + m_i(w_i, \mathbb{V}_{-i})$

$= \underbrace{v_i \cdot \mathbb{1}_{\{w_i + \sum_{j \neq i} v_j \geq 0\}}}_{\text{function of } v_i \text{ and } w_i} + \underbrace{\left[\sum_{j \neq i} v_j \cdot \mathbb{1}_{\{w_i + \sum_{j \neq i} v_j \geq 0\}} - \sum_{j \neq i} v_j \cdot \mathbb{1}_{\{\sum_{j \neq i} v_j \geq 0\}}\right]}_{\text{function of } w_i}$

$m_i(w_i, \mathbb{V}_{-i}) = \begin{cases} 0 & \text{if you are } \underline{\text{not}} \text{ pivotal} \\ -\sum_{j \neq i} v_j & \text{pivotal, } \sum_{j \neq i} v_j \geq 0, \ \sum v_j + w_i < 0 \\ \sum_{j \neq i} v_j & \text{pivotal, } \sum_{j \neq i} v_j < 0, \ \sum v_j + w_i \geq 0 \end{cases}$

Suppose you are not pivotal $(\sum v_j + v_i \geq 0) \Rightarrow (v_i > \sum v_j)$

- pretend to be pivotal (eg $-100M$) $(\sum v_j \geq 0)$
  - $m_i(-100M, v_{-i}) = -\sum_{j \neq i} v_j \Rightarrow u = v_i \mathbb{1} - \sum_{j \neq i} v_j = \sum_{j \neq i} v_j < \underset{\text{truth}}{v_i}$ since $\mathbb{1} = 0$
- pretend to be pivotal (eg $+100M$) $(\sum v_j < 0, \ \sum v_j + v_i < 0)$
  - $v_i + m_i(+100M, v_{-i}) = v_i + [\sum_{j \neq i} v_j] < \underset{\text{(truth)}}{0}$

(2) Pivotal $(\Sigma v_j > 0,\ \Sigma v_j + v_i < 0)$
Tell truth $\Rightarrow 0 + [-\sum_{j\neq i} v_j] > v_i$ (since $\Sigma v_j + v_i < 0$)

Lie (100 M) $\Rightarrow v_i + \underbrace{[\sum_{j\neq i} v_j - \sum_{j\neq i} v_j]}_{0 \text{ since no longer pivotal}} = v_i$

Pivotal $\Rightarrow \Sigma v_j$ or $-\Sigma v_j$
Not $\Rightarrow 0$

Revenue $= \Sigma - \mathbb{1}_{\{(\sum_{j\neq i} v_j + v_i)(\sum_{j\neq i} v_j) < 0\}} |\Sigma v_j|$
loss of efficiency

Find connection between Clark-Grove, Park problem, Monopoly regulation
Q2 HW6 2004
b employer $e_b$
s employee $e_s$
$\pi(e_b, e_s)$ $V_{bs}$ - profit from assignment

$\overset{GFT}{V_{bs}(e_b, e_s)} = \pi(e_b, e_s) V_{bs} - [C_b(e_b) + C_s(e_s)]$

a) FOCs to maximize GFT?
$\pi_1(e_b, e_s) V_{bs} - C_b'(e_b) = 0 \Rightarrow \pi_1 V_{bs} = C_b'$
$\pi_2(e_b, e_s) V_{bs} - C_s'(e_s) = 0 \Rightarrow \pi_2 V_{bs} = C_s'$
} Gives FB allocation (first best)

b) $\lambda_b V_{bs}$ $\lambda_s V_{bs}$ $(\lambda_s = 1 - \lambda_b)$ profit sharing rule.
What is Nash Equilibrium?
$\pi(e_b, \bar{e}_s) \lambda_b V_{bs} - C_b(e_b) \Rightarrow \lambda_b \pi_1 V_{bs} = C_b'$
$\pi(\bar{e}_b, e_s) \lambda_s V_{bs} - C_s(e_s) \Rightarrow \lambda_s \pi_2 V_{bs} = C_s'$
} Not FB

c) $MP_b = V_{bs}(e_b, e_s) - V_{bs}(0, e_s)$. $MP_s = V_{bs}(e_b, e_s) - V_{bs}(e_b, 0)$
NE: (b) $\pi_1 V_{bs} - C_b' = 0$ If everyone gets MP
(s) $\pi_2 V_{bs} - C_s' = 0$
Optimal effort level is implemented in this setting
$MP_b + MP_s = 2\pi(e_b^*, e_s^*) V_{bs} - C_b(e_b^*) - C_s(e_s^*)$
$- \pi(e_b^*, 0) V_{bs} - C_s(0)$
$- \pi(e_s^*, 0) V_{bs} - C_b(0)$

$= \pi(e_b^*, e_s^*) - c_b(e_b^*) - c_s(e_s^*)$
$+ [\underbrace{\pi(e_b^*, e_s^*)v_{bs} - \pi(e_b^*, 0)v_{bs} - \pi(0, e_s^*)v_{bs}}_{\text{if this = 0, it can be self-financed}}]$

$= 0$ if $\pi(e_b^*, e_s^*) = \pi(e_b^*, 0) + \pi(0, e_s^*)$
eg: $\pi(e_b, e_s) = e_b + e_s$

d) Suppose $\lambda_b = \lambda_s = 1$ $\Rightarrow$ $\pi(e_b^*, e_s^*)v_{bs} - c_b(e_b^*) - c_s(e_s^*) + \overbrace{\pi(e_b^*, e_s^*)v_{bs}}^{\text{need this amt of money}}$
$MP_b$, $MP_s$ $\Rightarrow$ $\pi(e_b^*, e_s^*) - c_b(e_b^*) - c_s(e_s^*)]$
$+ \underbrace{[\pi(e_b^*, e_s^*) - \pi(e_b^*, 0) - \pi(0, e_s^*)]v_{bs}}_{\text{need this amt of money.}}$

Thus, the second scheme is better.

e) $t_i + t_j = \pi(e_b^*, e_s^*)v_{bs}$
└ self-financing scheme
Pay $t_i$ and $t_j$ to headhunter

Mechanism:

IC - incentive compatibility
IR - individual rationality
BB - budget balancing
EF - efficiency

} Impossible in finite economy

In large economy, we can achieve all these if we give everyone their marginal productivity