Externalities (Real - directly through the v)
- Incomplete vs. Complete markets
- Coase Theorem

Public Goods (Excludable)
- Reversal of "prices" and "quantities"
- Costless vs costly public goods
- Free rider problem
- Intro. to mechanism design

Pricing in games.

$v_1(z_1, z_2)$ $v_2(z_1, z_2)$. n people $\Rightarrow n^2$ prices

Incomplete mkts: p

Complete mkts: $p_{11}, p_{12}, p_{21}, p_{22}$ - price that i pays for j's consumption.

Coase - The existence of p_{12} and p_{21} imply some sort of property rights.

an example of applicability of Coase thm: Mead's bee-keeper/apple example

L "Beehives for rent" in Davis.

"Property rights - taking behavior"

Public goods

In LE, there is a complete reversal of roles of prices and quantities.

L n people, ℓ commodities $\Rightarrow n \cdot \ell$ prices

$V_i(z) = v_i(z_i)\ \forall i$

V_0 - supplier of public good. $v_0(z) = -c_0(z)$

1) $v_0(z) = \begin{cases} 0 & \text{if } z \in [0, 1] \\ -\infty & \text{else} \end{cases}$ ← costless public goods.

2) $v_i : \mathbb{R}_+ \to \mathbb{R}_+$  ← costly public goods

$c_0 : \mathbb{R}_+ \to \mathbb{R}_+$ 

left = everyone is dumb; right = everyone is smart

Equilibrium: $v = (v_1, v_2, v_0)$ costless

$\hookrightarrow (z_1, z_2, z_0, p_1, p_2, p_0)$ satisfying

a) $\left.\begin{aligned} v_1(z_1) - p_1 z_1 &= v_1{}^*(p_1) \\ v_2(z_2) - p_2 z_2 &= v_2{}^*(p_2) \\ v_0(z_0) - p_0 z_0 &= v_0{}^*(p_0) \end{aligned}\right\}$ maximization

b) $z_1 = z_2 = z_0 \xleftrightarrow{\text{analog in private goods}} p_1 = p_2 = p$

$p_1 + p_2 = -p_0 = 0 \longleftrightarrow z_1 + z_2 = -z_0 = 0$

we will just assert this

Costly public goods:

$p_1 z_1 + p_2 z_2 = (p_1 + p_2) z_0 = c_0(z_0)$ if MC is constant

$p_i = MU_i = v_i'(z_0) \quad i \in \{1, 2\}$

L There is incentive to underreport $MU_i \ \forall i$

L Free rider problem

Free rider problem - can it be solved by an optimal mechanism?

	b_1	b_2
a_1		
a_2		

$x = (x_{11}, x_{12}, x_{21}, x_{22})$

$Pr(a_i, b_j) = x_{ij}$

$v_1 [(a_1, b_1), (a_1, b_2), (a_2, b_1), (a_2, b_2)]$

Public good problem is the choice of an x.

$v_1 \cdot x$ - payoff

5,1	0,0
4,4	1,5

problem: $\max \ v_1 \cdot x_1 + v_2 \cdot x_2 + v_0 \cdot x_0$ without IC constraints

assume $v_0 \cdot x = 0 \quad \forall x \geq 0$.

Define $p^1 = (p^1_{11}, p^1_{12}, p^1_{21}, p^1_{22})$, $p^2 = (p^2_{11}, p^2_{12}, p^2_{21}, p^2_{22})$

$V_i \cdot X = -\infty$ if X is not a probability distribution.

Equilibrium: $(x_1, x_2, x_0, p^1, p^2, p^0 = -(p^1+p^2))$ st.

$$v_1 \cdot x_1 - p^1 \cdot x_1 = v_1{*}(p^1)$$
$$v_2 \cdot x_2 - p^2 \cdot x_2 = v_2{*}(p^2)$$
$$v_0 \cdot x_0 - p^0 \cdot x_0 = v_0{*}(p^0)$$

$x_1 = x_2 = x_0$
$p^1 + p^2 = 0$ ← this condition might be superfluous

This will solve:

$$\max_{x_1, x_2, x_0} \; v_1 \cdot x_1 + v_2 \cdot x_2 + v_0 \cdot x_0 \quad \text{st} \quad x_1 = x_2 = x_0$$

Lindahl version of efficiency of a game.

	b_1	b_2
a_1	5, 1	0, 0
a_2	4, 4	1, 5

NE: $x_{11} = 1$ $\quad (x_{ij} = 1/4)$
$x_{22} = 1$

	b_1	b_2	
a_1	x_{11}	x_{12}	$\pi_1(a_1) = x_{11} + x_{12}$
a_2	x_{21}	x_{22}	$\pi_1(a_2)$
	$\pi_2(b_1)$	$\pi_2(b_2)$	

NE: $x_{11} = \pi_1(a_1) \cdot \pi_2(b_1)$, etc.
If there are incentive compatibility constraint:
How close to $(4,4)$ can we get? We will be able to get to $(3\frac{1}{3}, 3\frac{1}{3})$ when $x_{11} = x_{21} = x_{22} = 1/3$

deviation

$$\left.\begin{array}{l} \Delta v_1(d) \cdot x \le 0 \\ \Delta v_2(d) \cdot x \le 0 \end{array}\right\} \text{IC constraints.}$$

no profitable deviations