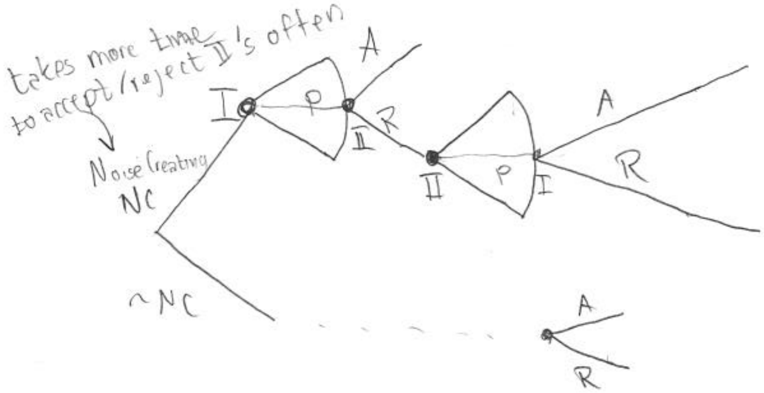
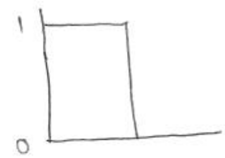
HW4, Q2

Externalities
└ complete vs. incomplete markets

Public goods
└ Lindahl equilibrium

HW4, Q2
"Perfect Competition and the Creativity of the Market"
! "Creating transactions costs"





Externalities
↳ $[(x_i),(y_j)] = a$

Define $a_{-i} = [(x_{-i}),(y_j)]$

$a_{-j} = [(x_i),(y_{-j})]$

$\succsim_i$ conditional on a_{-i} "my preferences depend on what everyone else is doing."

↳ $\succsim_i[a_{-i}] \rightarrow B_i(x_i; a_{-i}) \subset \mathbb{R}^\ell$

$Y_j(a_{-j})$

$\pi_j(p; a_{-j}) = \sup pY(a_{-j})$

$\xi_i(p, w_i(p); a_{-i})$

$EQ: [(x_i^*),(y_j^*)]$ p

1) $x_i^* \in \xi_i(p, w_i(p); a_{-i}^*)$ $\forall i$

2) $y_j^* \in \eta_j(p; a_{-j}^*)$ $\forall j$

3) $\sum x_i^* - \sum y_j^* = \omega$.

We will not have the first welfare theorem here.

Illustration: $v_1(z_1, z_2)$, $v_2(z_1, z_2)$, $z_1, z_2 \in \mathbb{R}$

v_i concave, $i \in \{1,2\}$

$(\pm)$? ← if this is "−", 1 envies 2. "+", 1 is altruistic towards 2.

1) Efficiency:

$$\underset{(+)}{\frac{\partial v_1}{\partial z_1}} + \frac{\partial v_2}{\partial z_1} = 0$$

$$\underset{(\pm)?}{\frac{\partial v_1}{\partial z_2}} + \underset{(+)}{\frac{\partial v_2}{\partial z_2}} = 0$$

} FOCs for $\max_{z_1, z_2} \{v_1(z_1, z_2) + v_2(z_1, z_2) ; z_1 + z_2 = 0\}$

"Incomplete" Markets

Looking for $(\bar{z}_1, \bar{z}_2)$, $p \in \mathbb{R}$ which satisfy:

1) $v_1^*(p; \bar{z}_2) = \sup_{z_1} \{v_1(z_1, \bar{z}_2) - p z_1\} = v_1(\bar{z}_1, \bar{z}_2) - p\bar{z}_1$

2) $v_2^*(p; \bar{z}_1) = \sup_{z_2} \{v_2(z_2, \bar{z}_1) - p z_2\} = v_2(\bar{z}_1, \bar{z}_2) - p\bar{z}_2$

and

3) $\bar{z}_1 + \bar{z}_2 = 0$

Without externalities, we solve this by:

$\max \{v_1(z_1) + v_2(z_2) ; z_1 + z_2 = 0\}$

gives us z_1^*, z_2^* $\Rightarrow p = v_1'(z_1^*) = v_2'(z_2^*)$

Each person pays $m_i = -p z_i^*$ $i \in \{1,2\}$

With $z_i \in \mathbb{R}^\ell$, $p = \nabla v_i(z_i^*)$

With incomplete mkts and externalities, there is no guarantee of efficiency.
- analogous to Cournot duopoly (taking others' behavior as fixed)

"Complete" Markets

Notation: z_i^j ← j name of person consuming; i ← name of person buying

$V_1(z_1^1, z_1^2) \equiv V_1(z_1, z_2)$

1 buys his own consumption; 1 buys 2's consumption

$V_2(z_2^1, z_2^2) \equiv V_2(z_1, z_2)$

prices: $p_1^1, p_1^2, p_2^1, p_2^2$

"Complete" Mkts equilibrium

$(\bar{z}_1^1, \bar{z}_1^2, \bar{z}_2^1, \bar{z}_2^2)$, $(\underbrace{p_1^1, p_1^2}_{\text{relevant to 1}}, \underbrace{p_2^1, p_2^2}_{\text{relevant to 2}})$ satisfying,

ideal tax/subsidies

1) $V(\bar{z}_1^1, \bar{z}_1^2) - [p_1^1 \bar{z}_1^1 + p_1^2 \bar{z}_1^2] = V_1^*(p_1^1, p_1^2)$

2) $V(\bar{z}_2^1, \bar{z}_2^2) - [p_2^1 \bar{z}_2^1 + p_2^2 \bar{z}_2^2] = V_2^*(p_2^1, p_2^2)$

3) $\left.\begin{cases} \bar{z}_1^1 = \bar{z}_2^1 \equiv \bar{z}^1 \\ \bar{z}_1^2 = \bar{z}_2^2 \equiv \bar{z}^2 \\ \bar{z}^1 + \bar{z}^2 = 0 \end{cases}\right\}$ Market clearing.

$\frac{\partial V_1}{\partial z_1} = p$ $\qquad \frac{\partial V_2}{\partial z_1} = ?$

$\frac{\partial V_1}{\partial z_2} = ?$ $\qquad \frac{\partial V_2}{\partial z_2} = p$

The magnitudes of ? emphasize the cost of not having complete markets.

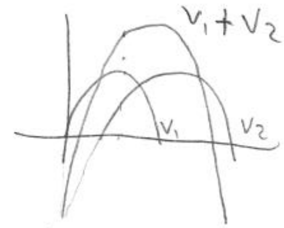
Internalizing externalities

Public Goods - Costless public good example

Want to choose $z \in [0,1]$ "shades of green between [0,1]"

$v_i(z)$ concave $\Rightarrow \Sigma v_i$ concave

Efficiency dictates: $\max_z [\Sigma_i v_i](z)$



- Is there a price-taking notion of equilibrium that would allow decentralization of efficient allocation.

  └ Lindahl Equilibrium

LE: $[\bar{z}_i]$, (p_i) ← personalized prices

1) $v_i(\bar{z}_i) - p_i \bar{z}_i = v_i^*(p_i) = \max_{z_i} \{v_i(z_i) - p_i z_i\}$

2) $\bar{z}_i = \bar{z} \quad \forall i$

3) $\sum_i p_i = 0$

This choice will be efficient:

$\Sigma v_i^*(p_i) = \Sigma v_i(\bar{z}) - \bar{z}\Sigma p_i = \Sigma v_i(\bar{z}) = \max_z [\Sigma v_i](z)$