average: 58
Median: 60
2/3 of the final is new stuff.

Under PCE, PT is optimal behavior.
2nd half of course:
i) mechanism design
ii) public goods and externalities
What is special about Rubinstein bargaining game that produces an efficient outcome?

2 models: I) $p \longrightarrow$ occupation $\longrightarrow z_i$
$\max U(\text{occupation}, z_i) - p z_i$
II) occupation $\longrightarrow p \longrightarrow z_i$
$\max u(\text{occupation}, z_i) - p(\text{occupation}) z_i$

2003 Final Q4: Lindahl Pricing
($\Rightarrow$ Public Good)

(z_i) p $\leftarrow$ private good economy
z (p_i) $\leftarrow$ public good economy

a) $u_i(z_i, z_j, m_i) = r_i \ln(z_i + z_j) + m_i$
$r_1 = 2$, $r_2 = 4$ cost of production: $c(z) = \sigma z$. Let $\sigma = 1$.
$V(0) = \max\{2\ln(z_1+z_2) + 4\ln(z_1+z_2) - (z_1+z_2)\}$
$= \max\{6\ln(z_1+z_2) - (z_1+z_2)\}$
(z_1): $\frac{6}{z_1+z_2} = 1 \Rightarrow z_1 + z_2 = 6$

$V_I(0) = 6\ln 6 - 6$

b) $MC = AC = 1 \Rightarrow p = 1$
Consumer: $\max\{r_i \ln(z_1+z_2) - z_i\}$
$\frac{r_i}{z_i+z_j} - 1 = 0 \Rightarrow \frac{2}{z_1+z_2} \leq 1$; $\frac{4}{z_1+z_2} \leq 1$

$\Rightarrow z_1 + z_2 = 4$. Not efficient.

c) Lindahl equilibrium

Private: $z_1 + z_2 = z_p$

$p = p$

Lindahl:

$p_1 + p_2 = p$

$C(y_1, y_2) = \sigma \max\{y_1, y_2\}$

Lindahl Pricing

Private $v_i(z_i)$ $p \cdot z_i$ $p_k^i = p_k \ \forall i$

Public $v_i(z)$ $p_i \cdot z$ $\sum_i p_k^i = p_k$

Ind: Private: $MU_k^i = p_k$ SP: Private: $\sum z_i = z^p$

Ind: Public: $MU_k^i = p_k^i$ SP: Public: $z = z^p$

Comp 2001S Q6:

$\underbrace{x \in [0,1]}$

botanical garden & sports field

$\text{Cost}(x) \equiv \text{constant} \quad \forall x$

$V_i = a_i x - b_i x^2 \qquad i = 1, \ldots, n$

a) $\sum_i [a_i x - b_i x^2] = x \sum a_i - x^2 \sum b_i$

FOC: $\sum a_i = 2x \sum b_i$

Define: $A = \sum a_i$, $B = \sum b_i$

Efficient: $x = \frac{A}{2B}$

b) $u = a_i x - b_i x^2 - p_i x$

FOC: $a_i - 2b_i x - p_i = 0 \Rightarrow x = \frac{a_i}{2b_i} - \frac{p_i}{2b_i}$

$\sum p_i = p^p \underset{\text{by assump.}}{=} 0$

$\Rightarrow p_i = a_i - 2b_i x$

$\sum p_i = 0 \Rightarrow \sum(a_i - 2b_i x) = 0$

$\Rightarrow A - 2Bx = 0$

$\Rightarrow x = \frac{A}{2B}$

c) $V_I(0) = \max\{x \sum a_i - x^2 \sum b_i\}$

$= \max\{Ax - Bx^2\}$

$= \frac{A^2}{2B} - \frac{A^2}{4B} = \frac{A^2}{4B}$

$V_{-i}(0) = \max_x [A_i x - B_i x^2]$ where $A_i = \sum_{j \neq i} a_j$, $B_i = \sum_{j \neq i} b_j$

$= \frac{A_i^2}{4B_i}$

$MP_I = \frac{A^2}{4B} - \frac{A_i^2}{4B_i}$

$$\left[\begin{array}{l} V_i = a_i x - b_i x^2 \\ P_i = a_i - 2b_i x \\ \quad = a_i - 2b_i \frac{A}{2B} \\ \\ U = a_i \frac{A}{2B} - b_i \left(\frac{A}{2B}\right)^2 - \left(a_i - 2b_i \frac{A}{2B}\right)\frac{A}{2B} = b_i \left(\frac{A}{2B}\right)^2 \end{array}\right]$$

$a_i x - b_i x^2 - \left[\hat{a}_i - 2\hat{b}_i \frac{A}{2B}\right] x$

— Break

c) $a_i\left(\frac{\hat{A}}{2B}\right) - b_i\left(\frac{\hat{A}}{2B}\right)^2 + \left[\frac{\hat{A}^2}{4B} - \frac{A_i^2}{4B_i}\right] - \hat{a}_i\left(\frac{\hat{A}}{2B}\right) + b_i\left(\frac{\hat{A}}{2B}\right)^2$ $\quad A = \sum_j a_j$, $\hat{A} = \sum_{j \neq i} a_j + \hat{a}_i$

$\frac{\partial}{\partial \hat{a}_i}$: $\frac{a_i}{2B} - \frac{2b_i \hat{A}}{4B^2} + \frac{2\hat{A}}{4B} - \frac{\hat{A}}{2B} - \frac{\hat{a}_i}{2B} + \frac{2b_i \hat{A}}{4B^2} = 0$

$2a_i B - \cancel{2b_i}\hat{A} + 2\hat{A}B - 2\hat{A}B - 2\hat{a}_i B + \cancel{2b_i}\hat{A} = 0$

$\Rightarrow a_i = \hat{a}_i \Rightarrow$ Tell the truth

d) Lindahl } same x, different money payments.
M (CI()

$m_i^L = -\left(a_i - 2b_i \frac{A}{2B}\right)\frac{A}{2B}$

$m_i^M = \left[\frac{A^2}{4B} - \frac{A_i}{4B_i}\right] - a_i\left(\frac{A}{2B}\right) + b_i\left(\frac{A}{2B}\right)^2$

$$-(m_i^L - m_i^M) = -\left[b_i\left(\frac{A}{2B}\right)^2 - \left(\frac{A^2}{4B} - \frac{A_i}{4B_i}\right)\right] \neq 0.$$

"cost of IC".

$\sum_i [-(m_i^L - m_i^M)]$ - total cost to designer of mechanism.

Honey Prod: $\pi(h) = 2h - \frac{h^2}{100} \Rightarrow h = 100$
Apple Prod: $\pi(a) = 3a - \left(\frac{a^2}{100} - h\right) \Rightarrow a = 150$ } comp. outcome.

$\max\{\pi(h) + \pi(a)\} \Rightarrow h = 150, a = 150$ } SP outcome.

$\pi(h) = 2h - \frac{h^2}{100} + sh \Rightarrow 2 - \frac{h}{50} + s = 0$

Want $h = 150 \Rightarrow 2 - \frac{150}{50} + s = 0 \Rightarrow s = 1$

2004F Q5
2002 F Q4 } Real vs. pecuniary.