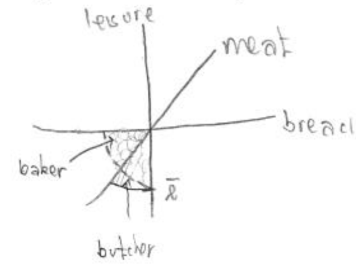
$\mathcal{E} = \{ (U_i, \omega_i, V_i) \}$ $\qquad v_i \in V_i \rightarrow Y_i(v_i)$ "each v_i defines technology"

$\mathbb{R}^\ell$ - "super" space

$\sup_{y_i \in Y_i(v_i)} U_i(\omega_i + y_i + z \mid v_i) \equiv v_i(z)$

"best way of attaining the trade 'z'"



$$\begin{array}{l} \text{leisure} \\ \text{bread} \\ \text{meat} \end{array} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1/2 \\ -1 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 1/2 \\ 0 \\ -1 \end{pmatrix}$$

no specialization — make bread (baker) — make meat (butcher)

Define $U_i^*(p \mid v_i) = \sup_z \left\{ \sup_{y_i \in Y_i} U_i(\omega_i + y_i + z \mid v_i) - pz \right\}$

$$= \sup_z v_i(z) - pz$$

$$U_i^*(p) = \sup_{v_i} \{ U_i^*(p \mid v_i) \}$$

PTE for $\mathcal{E}$: $\quad p \in \mathbb{R}^\ell \quad (z_i) \quad (m_i)$ such that $\qquad$ "in PTE prices proceed maximization"

i) $\Sigma z_i = 0$

ii) $U_i^*(p) = U_i(\omega_i + y_i + z_i \mid v_i) - pz_i = v_i(z_i) - pz_i$

Proposition: PTE for $\mathcal{E}$ is efficient.

Pf: $\forall p,\ U_i^*(p) \underset{\forall v_i}{\geq} U_i^*(p \mid v_i) \underset{\forall z_i}{\geq} v_i(z_i) - pz_i$

By market clearing, $\Sigma z_i = 0$

Thus, $\forall p,\ \Sigma U_i^*(p) \geq \Sigma v_i(z_i)$

Therefore $\Sigma U_i^*(p) \geq g(v)$ $\qquad$ where $g(v) = \sup\{\Sigma v_i(z_i) : \Sigma z_i = 0\}$

By PTE, we have achieved: $\Sigma U_i^*(p) = \Sigma v_i(z_i) - p\Sigma z_i = \Sigma v_i(z_i)$

Thus, $\Sigma v_i(z_i) \geq g(v) \Rightarrow \Sigma v_i(z_i) = g(v) \rightarrow$ PO $\square$

Revise the link between prices and maximization:

"Leading to" $\rightarrow$ Instead of $p \rightarrow \mathbb{V} \rightarrow (z_i)$, we want $\mathbb{V} \rightarrow \hat{p} \rightarrow (\hat{z}_i)$

Two-stage game:

Stage 1: Choose $\mathbb{V}$

Stage 2: Payoffs are determined as $\hat{\pi}_i(\mathbb{V}) = \pi_i(p(\mathbb{V})) = v_i^*(p(\mathbb{V}))$

L There are no actions here, so it is effectively one-stage.

$((\hat{\pi}_i), V)$ is a game in normal form $[V = V_1 \times \ldots \times V_n]$

$\mathbb{V} \xrightarrow{\text{determines}} \ell(\mathbb{V})$ $\qquad H(\mathbb{V}) = \{c : z_{ic} < 0 \text{ for some } v_i(z_i) > -\infty\}$

$\mathbb{R}^{\ell(\mathbb{V})} \subset \mathbb{R}^{\ell}$

Equilibrium: $\hat{\pi}_i(\mathbb{V}) \geq \hat{\pi}_i(v_{-i}, v_i') \quad \forall i, \forall v_i'$

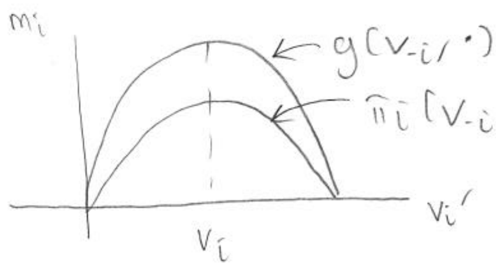
Efficiency wrt $\mathbb{V}$ $(\mathbb{R}^{\ell(\mathbb{V})})$ (when $\mathbb{R}^{\ell(\mathbb{V})} = \mathbb{R}^{\ell}$)

$\dfrac{\Delta \hat{\pi}_i(\mathbb{V})}{\Delta v_i} = \hat{\pi}_i(v_{-i}, v_i') - \hat{\pi}_i(\mathbb{V})$

$\dfrac{\Delta g(\mathbb{V})}{\Delta v_i} = g(\mathbb{V}, v_i') - g(\mathbb{V})$

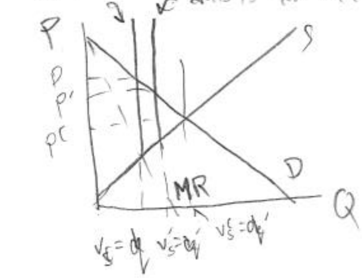
Is it the case that $\overset{(-)}{\dfrac{\Delta \hat{\pi}_i(\mathbb{V})}{\Delta v_i}} \overset{(-)}{\dfrac{\Delta g(\mathbb{V})}{\Delta v_i}} \geq 0$ $\leftarrow$ This is what Adam Smith "said"

Problem: We have non-efficiency if $\Delta\hat{\pi}_i(\mathbb{V}) \Delta g(\mathbb{V}) < 0$



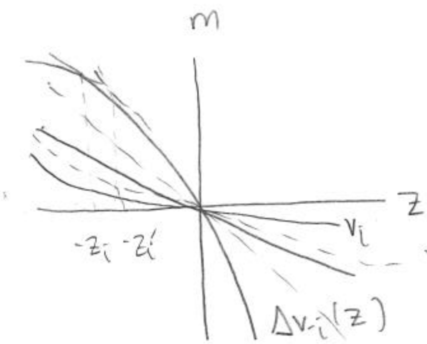
This is a problem.
$\Delta\hat{\pi}_i(\mathbb{V}) \Delta g(\mathbb{V}) < 0$

equilibrium. This is an occupational choice.



"Can I change my characteristics in a favorable way?

- Misrepresentation of preferences in order to get a more favorable prices

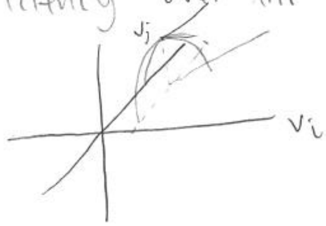


v_i' ← not true preferences (reported)

new, more favorable prices for you.

This would not occur under perfect competition. $(\Delta\pi_i(\mathbb{v}) = \Delta g(\mathbb{v}))$

Efficiency over all $\mathbb{v}$



$\Delta\pi_i(\mathbb{v}) = \Delta g(\mathbb{v})$ (FA)

It is possible that

$\frac{\Delta g(\mathbb{v})}{\Delta v_i} < 0 \ \forall i$, but $\frac{\Delta g(\mathbb{v})}{\Delta \mathbb{v}} > 0$

$\tilde{p} \in \mathbb{R}^{\ell}$ ← let $\tilde{p}$ be the price that "precedes maximization"

It would be nice if $\tilde{p} = (p_c)$

$$\ell \left\{ \begin{bmatrix} p_c^i \\ p_c \\ p_c^j \\ \ \end{bmatrix} \right. \begin{array}{l} c \in H(\mathbb{v}_{-i}, v_i') \setminus H(\mathbb{v}) \text{ "new commodities that } i \text{ could produce"} \\ c \in H(\mathbb{v}) \\ c \in H(\mathbb{v}_{-j}, v_j') \setminus H(\mathbb{v}) \text{ "new commodities that } j \text{ could produce"} \end{array}$$

This $\tilde{p}$ is constucted without regards to coordination problem