General equilibrium and game theory

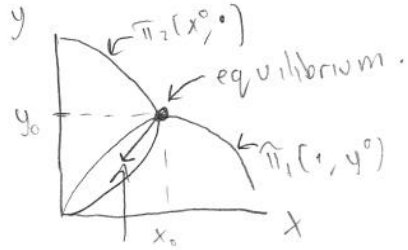
2 person game:

$\pi_1(x, y)$, $\pi_2(x, y)$ ← payoffs ($\exists$ externalities since $\pi_1 : \Sigma_1 \times \Sigma_2 \to \mathbb{R}$)

1's strat. 2's strat

Nash equilibrium: $\dfrac{\partial \pi_1(x^\circ, y^\circ)}{\partial x} = \dfrac{\partial \pi_2(x^\circ, y^\circ)}{\partial y} = 0$ ← FOC_2

(does not take externalities into account)



move toward "efficiency" if collude.

takes into account impact on the other firm

Cartel:
$$\frac{\partial \pi_1(x,y)}{\partial x} + \frac{\partial \pi_2(x,y)}{\partial x} = 0$$
$$\frac{\partial \pi_1(x,y)}{\partial y} + \frac{\partial \pi_2(x,y)}{\partial y} = 0$$
} "efficient" for the firms (takes externalities into account.)

Introduce strategic behavior.

$\mathbb{v} = (v_1, \dots, v_n)$ Players "choose" their v_i (occupations)

$\pi_i(v_1, \dots, v_n)$ - payoffs

$\equiv \pi_i(\mathbb{v})$

$\pi_i(\mathbb{v}_{-i}, v_i')$ - i has deviated from v_i to v_i'

a NE is a $\mathbb{v}$ $\ni$ $\pi_i(\mathbb{v}) \geq \pi_i(\mathbb{v}_{-i}, v_i')$ $\forall i \forall v_i'$

[Before, efficiency (for a fixed $\mathbb{v}$) was $V_I(0)$.]

$\pi_i : V_1 \times V_2 \times \dots \times V_n \to \mathbb{R}$

$\mathbb{v} \in V_1 \times \dots \times V_n \equiv V$ $\quad g(\mathbb{v}) = \max\{\Sigma v_i(z_i) : \Sigma z_i = 0\}$

Is it true that $\Sigma \pi_i(\mathbb{v}) = g(\mathbb{v})$

Want to choose $\max_{\mathbb{v} \in V} g(\mathbb{v})$

Note: z_i is a function of $\mathbb{v}$.

$\mathbb{v} \to g(\mathbb{v}) = \max\{\sum v_i(z_i) : \sum z_i = 0\}$

$\sum_i v_i(z_i(\mathbb{v})) = g(\mathbb{v})$

Is $\mathbb{v} \ni \pi_i(\mathbb{v}) \geq \pi_i(\mathbb{v}_{-i}, v_i') \ \forall i \iff \mathbb{v} \in \arg\max g(\mathbb{v})$?

(ie, are equilibria efficient?)

$v_i \in V_i$

$U_i : \mathbb{R}^\ell_+ \to \mathbb{R}$ "grand utility fcn" $\quad \omega_i \in \mathbb{R}^\ell_+$

$v_i(z_i) \equiv v_i(z_i \mid Y_i) = \sup_{y_i \in Y_i} U_i(\omega_i + y_i + z_i)$

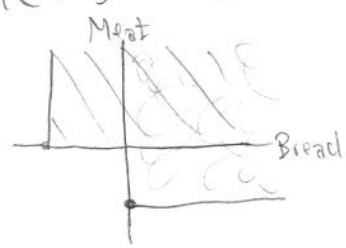
* Once you pick your occupation, you choose a portion of your "grand utility function"
↳ You are not choosing your utility function.

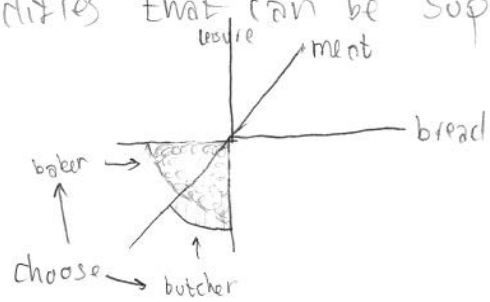
2 stage game:

1) Pick occupations $\mathbb{v}$
2) Maximize utility in a PTE.

The focus will be on the first stage.

$H(\mathbb{v})$: the commodities that can be supplied from $\mathbb{v}$.





can choose ||| or ⊗

Suppose there is no muffins in $H(\mathbb{v})$. Then it may be profitable to bake muffins

$\mathbb{R}^{\ell(\mathbb{v})}$ - commodity space $= \{z \in \mathbb{R}^\ell : z_c = 0 \ \forall c \notin H(\mathbb{v})\}$

it is typically the case that $|H(\mathbb{v})| << \ell$.

a social planner would find $g(\mathbb{v}) \ \forall \mathbb{v}$ and then take the maximum of all these.

2nd stage

$\mathbb{V} \rightarrow$ selection of PTE for $\mathbb{V}$. (everyone knows $p(\mathbb{V}, v_i') \ \forall v_i'$)

$z_i(\mathbb{V}) \rightarrow p(\mathbb{V}) \leftarrow$ prices

$$\pi_i(\mathbb{V}) = v_i^*(p(\mathbb{V})) = v_i(z_i(\mathbb{V})) - p(\mathbb{V}) z_i(\mathbb{V})$$

$$\equiv \sup_{z_i \in \mathbb{R}^{\ell(\mathbb{V})}} v_i(z_i) - p(\mathbb{V}) z_i$$

$$\left[\sum \pi_i(\mathbb{V}) = \sum v_i^*(p(\mathbb{V})) = g(\mathbb{V}) \ \forall \ \mathbb{V} \in V \right]$$

First welfare thm

$\pi_i(\mathbb{V}) = v_i^*(p(\mathbb{V})) \leftarrow$ payoff structure

Real externality: someone does something <u>outside</u> the price system that affects prices.

Pecuniary: Someone does something which affects prices which affects others' utilities.

Ostroy: Pecuniary externalities are important. (caused by lots of people)

Landlord story.

$p(\mathbb{V}) = p(\mathbb{V}_i, V_{-i}) \quad \forall i \Rightarrow$ competitive markets

$p(\mathbb{V}_{-j}, V_j') = p(\mathbb{V}) \Rightarrow$ No external affects.

Break

$\pi_i(\mathbb{V}) = V_i^*(p(\mathbb{V}))$

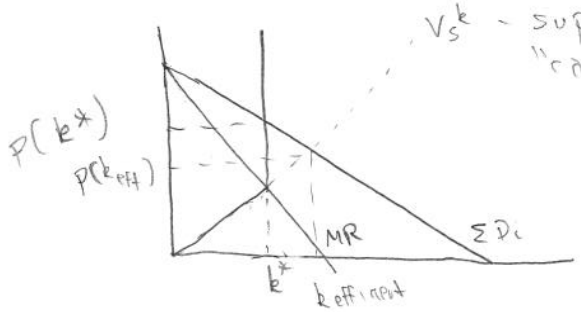
if $\pi_i(\mathbb{V}_{-j}, v_j') \neq \pi_i(\mathbb{V}) \rightarrow$ externality. (j deviates and affects i)

↳ It doesn't matter what the source of the externality is according to game theory.

$I = \{1, \ldots, n\}$

$V_b = az - \frac{1}{2}z^2$

V_s^k - supply: can choose k "capacity"



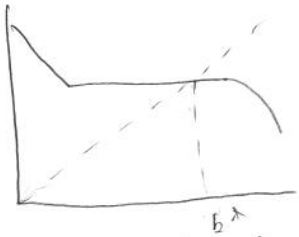
$\pi_b(k) - p(k)$

$\pi_s(k) \geq \pi_s(k') \ \forall k'$

Occupational equilibrium which is not efficient.

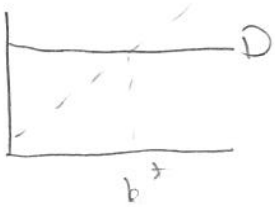
There are externalities in the form of pecuniary externalities. (CS)

Suppose there are flat spots in the u-fcn:



$\Rightarrow$ This will give us perfect competition.

Suppose n (# buyers) $\to \infty$



$\Rightarrow$ This will give us perfect competition.

As soon as you affect prices there is a problem.

$\pi_i(v) - \pi_i(v_{-i}, v_i')$ $\to$ private marginal product

$g(v) - g(v_{-i}, v_i')$ $\to$ social marginal product

Those are equal under perfect competition.