Decentralized Knowledge?

$\pi_i(\mathbb{v}) = \pi_i(p(\mathbb{v}))$ $\mathbb{v}$ affects my payoff <u>only</u> through market prices

$$\frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'} \equiv \pi_i(\mathbb{v}) - \pi_i(\mathbb{v}_{-i}, v_i') \equiv PMP_i$$

$$\frac{\Delta g(\mathbb{v})}{\Delta v_i'} \equiv g(\mathbb{v}) - g(\mathbb{v}_{-i}, v_i') \equiv SMP_i$$

NE: $\frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'} \leq 0 \quad \forall v_i' \ \forall i$

Monopoly equilibrium: NE: $\frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'} \leq 0$ and $\frac{\Delta g(\mathbb{v})}{\Delta v_i'} > 0$

or $\frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'} \cdot \frac{\Delta g(\mathbb{v})}{\Delta v_i'} < 0$

FA: $$\underbrace{\frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'}}_{PMB_i} = \underbrace{\frac{\Delta g(\mathbb{v})}{\Delta v_i'}}_{SMB_i}$$

Limited notion of efficiency:

$\frac{\Delta g(\mathbb{v})}{\Delta v_i'} \leq 0 \quad \forall v_i' \ \forall i$ "any one-person change would not be desirable."

Proposition: NE + FA $\Rightarrow \frac{\Delta g(\mathbb{v})}{\Delta v_i'} \leq 0 \quad \forall v_i' \ \forall i$

Pf: $0 \geq \frac{\Delta\pi_i(\mathbb{v})}{\Delta v_i'} = \frac{\Delta g(\mathbb{v})}{\Delta v_i'} \quad \forall v_i' \ \forall i \quad \square$.

What is the economic interpretation of this?

World with standardized commodities: $\ell(\mathbb{v}) = \ell \quad \forall \mathbb{v}$.

$\pi_i(\mathbb{v}) = \pi_i(p(\mathbb{v}))$

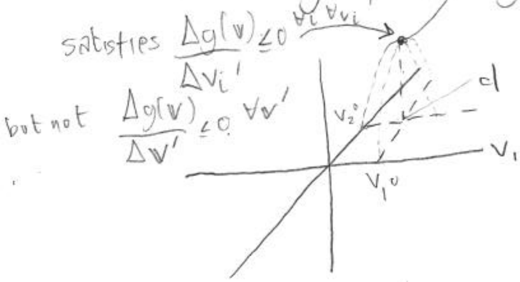
$p(\mathbb{v}) = p(\mathbb{v}_{-i}, v_i') \quad \forall v_i'$ (PEDS)

* No commodity innovation + PEDS $\Rightarrow$ FA

$\frac{\Delta g(v)}{\Delta v_i'} \le 0 \quad \forall v_i' \quad \forall i \overset{?}{\Rightarrow} g(v) = \max_{v' \in V} g(v')$ No.

There may $\exists\, v' = (v_1', \ldots, v_n')$ such that $\frac{\Delta g(v)}{\Delta v'} > 0$ ← multidimensional change.

i.e. $g(v') > g(v)$ $\quad \left[\frac{\Delta g(v)}{\Delta v'} \equiv g(v') - g(v) \right]$



can gain if move in d direction.

Coordination problems - cannot occur without commodity innovation.

No complements assumption rules this out:

$\frac{\Delta g(v)}{\Delta v_i'} \le 0 \ \forall i \ \forall v_i'$

$\Rightarrow \sum \frac{\Delta g(v)}{\Delta v_i'} \le 0$ but $\frac{\Delta g(v)}{\Delta v'} > \sum_i \frac{\Delta g(v)}{\Delta v_i'}$

NC $\Rightarrow (\le)$

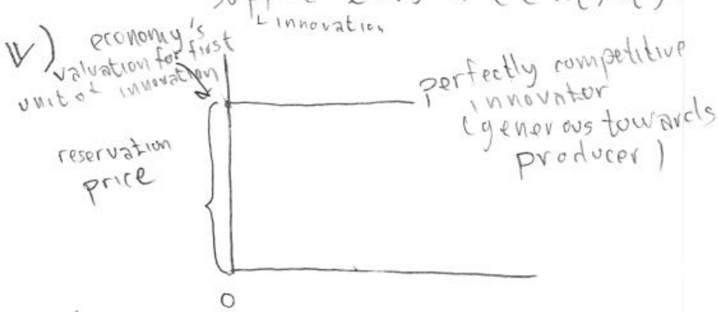
$NE + FA \not\Rightarrow \frac{\Delta g(v)}{\Delta v'} \le 0 \quad \forall\, v'$

$NE + FA + NC \Rightarrow \frac{\Delta g(v)}{\Delta v'} \le 0 \quad \forall\, v'$ Efficiency.

Commodity innovation: $H(v)$ suppose $\ell(v) < \ell(v_{-i}, v_i)$ ← innovation

$p \in \mathbb{R}^{\ell(v)}$ vs $\hat{p} \in \mathbb{R}^{\ell}$

Hayek is referring to this price system



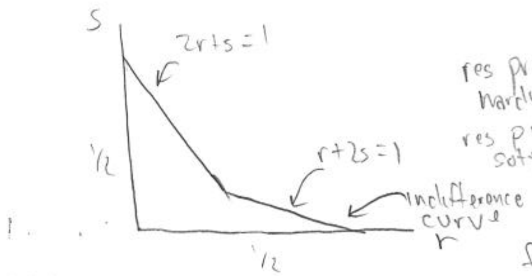
I know $p \in \mathbb{R}^{\ell(v)}$ plus some knowledge about expanding in certain directions, but I do not know $\hat{p} \in \mathbb{R}^{\ell}$

This is a world of decentralized innovation. Each person has some localized information about prices in markets they can innovate, but no one knows the big picture.

$\min\{2r + s,\ r + 2s\}$ (r = hardware, s = software)

Neither s nor r exists right now

$1_r = (1, 0)$ $\quad 1_s = (0, 1)$



res price for hardware → $Dv(0; 1_r) = 1$

res price for software → $Dv(0; 1_s) = 1$

res price for package → $Dv(0; 1_r + 1_s) = 3$

} complimentary commodities

Suppose MC of innovate $r = 1.25$
Suppose MC of innovate $s = 1.25$
} Not privately profitable to innovate

$\Rightarrow$ It is socially profitable.

1st theorem would have known $Dv(0; 1_r + 1_s) = 3$.
└ this is not very decentralized.

NC: additivity of directional derivative (smoothness)
└ defn of differentiability is additivity of directional derivatives.

Concavity $\Rightarrow$ superadditivity of directional derivatives

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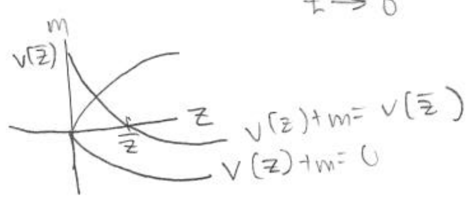
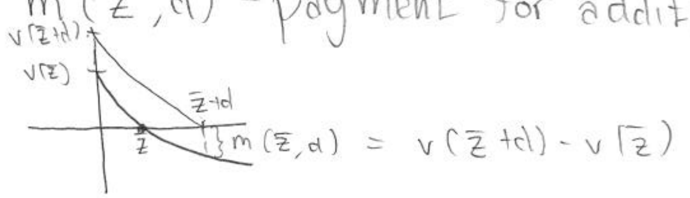
$v(z) + m$

(a) $Dv(\bar{z}; d) = \lim_{t \to 0} \dfrac{v(\bar{z} + td) - v(\bar{z})}{t}$

(b)



(c) $m(\bar{z}, d)$ - payment for additional d at $\bar{z}$.



$m(\bar{z}, d) = -[v(\bar{z} + td) - v(\bar{z})]$

d) 2 identical individuals

$m_2(\bar{z}; d) = \max\{ m(\bar{z}; d_1) + m(\bar{z}; d_2) \quad st \quad d_1 + d_2 = d \}$

$= \max\{ -v(\bar{z} + d_1) + v(\bar{z}) - v(\bar{z} + d - d_1) + v(\bar{z}) \}$

occurs when $d_1 = d_2 = \frac{d}{2}$

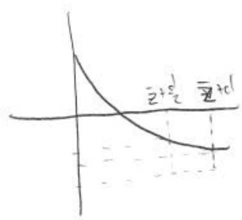
convexity

$= -2v(\bar{z} + \frac{d}{2}) + 2v(\bar{z}) < -2\left[\frac{v(\bar{z}) + v(\bar{z}+d)}{2}\right] + 2v(\bar{z})$

since $2v(\bar{z} + \frac{d}{2}) > v(\bar{z} + d)$

$= -v(\bar{z}+d) + v(\bar{z})$

$= m(\bar{z}; d)$



e) $m_k(\bar{z}; d) = k \cdot m(\bar{z}; \frac{d}{k})$

$= k\left[-v(\bar{z} + \frac{d}{k}) + v(\bar{z})\right]$

$= -\dfrac{v(\bar{z} + d/k) - v(\bar{z})}{\frac{1}{k}} \xrightarrow{k \to \infty} -Dv(\bar{z}; d)$

f) Flattening effect