$\underline{v} = (v_1, \dots, v_n)$

$h(r, \bar{z}) = \max \{ \sum_i v_i(z) x(i,z) : (*) \quad x(i,z) \geq 0 \}$ - continuum economy

$h^*(r, \bar{z}) = \max \{ \sum_i v_i(z) x(i,z) : (*) \quad x(i,z) \in \{0,1,2,\dots\} \}$ - finite individual,

and $x(i,z)$ must be feasible $(*)$

$$\sum_z x(i,z) = r_i, \quad x(z) \equiv \sum_i x(i,z) \Rightarrow \sum_z z\, x(z) = \bar{z}$$

Obviously, $h^*(r, \bar{z}) \leq h(r, \bar{z})$

Let

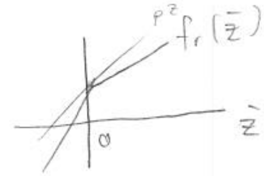
$g(r) \equiv h(r, 0)$ and $f_r(\bar{z}) = h(r, \bar{z})$

$g^*(r) = h^*(r, 0)$ and $f_r^*(\bar{z}) = h^*(r, \bar{z})$

focus on people — focus on commodities

Suppose $\{p\} = \partial f_r(0) = \nabla f_r(0) = \left(\frac{\partial f_r(0)}{\partial z_i} \right)_{i=1}^n$

We need not assume differentiability : $p \in \partial f_r(0)$



$\{q\} = \partial g(r) = \nabla g(r) = \left(\frac{\partial g(r)}{\partial r_i} \right)_{i=1}^n$

$\left[\text{PTE} : (x, p) \Rightarrow x(i,z) > 0 \Rightarrow -pz = m \text{ and } \underset{v_i^*(p)}{\underset{\parallel}{v_i(z) + m}} \geq v_i(z') - pz' \ \forall z' \right]$

$q_i = v_i^*(p)$

$\frac{\partial g(r)}{\partial r_i} = MP_i$ In a continuum economy, $\frac{\partial g(r)}{\partial r_i} = v_i^*(p) \forall i$

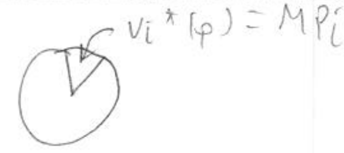
[Euler's theorem: $\Psi(y)$ homogeneous of degree 1, differentiable

$\Rightarrow \underbrace{\nabla \Psi(y) \cdot y}_{\text{payments based on marginal product}} = \underbrace{\Psi(y)}_{\text{total product}} \Rightarrow$ No profit in CRS economy]

g is homogeneous of degree 1 with respect to r due to concavifying effect of large numbers.

$\Rightarrow \nabla g(r) \cdot r = g(r)$ by Euler's theorem.

In gen'l 

but in this case 

$p_i = \dfrac{\partial f_r(0)}{\partial z_i}$ - rate at which pie increases when you increase z_i

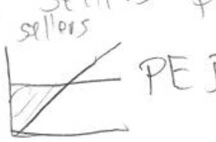
$q_i = \dfrac{\partial g(r)}{\partial r_i}$ - rate at which pie increases when you increase r_i

[$v_I(0) = h^*(\underline{1}, 0)$]

[$MP_i = v_I(0) - v_{I\setminus\{i\}}(0) = h^*(\underline{1}, 0) - h^*(\underline{1} - 1_i, 0)$]

In order to get $MP_i = v_i^*(p)$ in the finite economy, you need flats

Buyers and sellers pretend to see:

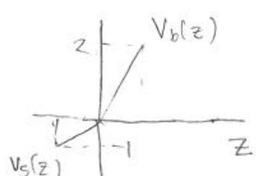


Master and servant problem

$v = (v_b, v_s)$, $r = (1,1)$ $\ell = 1$

$v_b(z) = 2z_b$ $0 \le z \le 1$

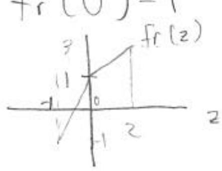
$v_s(z) = z$ $-1 \le z \le 0$



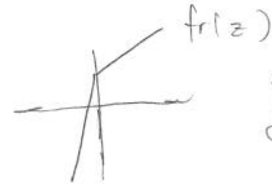
$p \in [1,2] \Rightarrow v_b^*(p) = v_b(1) - p \cdot 1 = 2 - p$

$v_s^*(p) = v_s(-1) - p(-1) = 1 + p$

$f_r(0) = 1$ $f_r(\frac{1}{2}) = \frac{1}{2}$



If $r \neq (1,1)$



the flattening effect does not occur here

Everyone has the ability to change prices.

└ Large numbers do not get you closer to perfect competition.

This is a knife edge case. Does not occur in $r = (16, 17)$ or $r = (17, 16)$. Unique prices here.

$q^*(r)$

$$\lim_{k \to \infty} \frac{g^*(k \cdot 1)}{k} = \infty$$

$g(r)$ $\quad T \subseteq I = \{1, \dots, n\}$

$V(T)$ - gains associated with the group T. (coalition)

game in characteristic function form

$V(\emptyset) \equiv 0$

$V(S \cup T) \geq V(S) + V(T)$ when $S \cap T = \emptyset$ superadditivity

$V(I)$ - size of pie

Let $q = (q_1, \dots, q_n) \ni \sum q_i = V(I)$

Clearly, $q_i \geq V(\{i\})$

Core

$G = (V, I) \qquad V: 2^I \to \mathbb{R}$

$\sum_i q_i = V(I)$

$\sum_{i \in T} q_i \geq V(T) \quad \forall T$. No credible threat by any coalition to withdraw.

must get more cooperating from than from not

$MP_i = V(I) - V(I \setminus \{i\})$

If q belongs to $Core(V, I) \Rightarrow q_i \leq MP_i$ by superadditivity

There is always a core in 2 players: $V(1,2) \geq V(1) + V(2)$)

Suppose $V(i) = 0 \qquad V(1,2,3) = 1$

$V(2,3) = \alpha$

Claim: $\alpha \leq 2/3 \Rightarrow \exists$ core

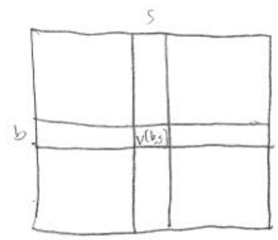
$\alpha > 2/3 \Rightarrow \nexists$ core $\qquad$ Verify this.

assignment model:

$I = B \cup S$

$V(b,s) \geq 0 \qquad V(b,b) = V(S,S) = 0$

Cannot assign more than one buyer to a seller and vice versa.



$$g(\underline{1}) = \max \sum_{b,s} V(b,s)x(b,s)$$

$$\text{st} \quad \sum_b x(b,s) \leq 1 \quad \forall s \qquad x(b,s) \geq 0$$

fractional matches.

$$\sum_s x(b,s) \leq 1 \quad \forall b$$

$x(b,s) \equiv$ mass of matches b/t types b and s.

$$g^*(\underline{1}) = \max \sum_{b,s} V(b,s)x(b,s)$$

$$\text{st} \quad x(b,s) \in \{0,1,2,\dots\}$$

$$\sum_b x(b,s) \leq 1 \quad \forall s$$

$$\sum_s x(b,s) \leq 1 \quad \forall b$$

Remarkable property of assignment model: $g(\underline{1}) = g^*(\underline{1})$

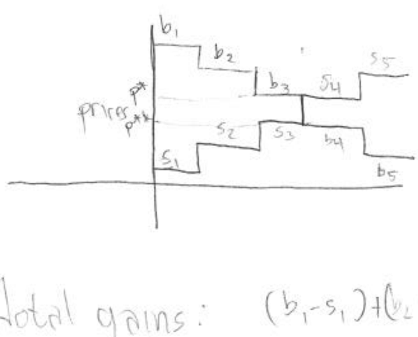
Perfect competition:

$$g^*(\underline{1}) - g^*(\underline{1} - \underline{1}_i)$$

Core for assignment model

$$(q_b, q_s) \ni q_b + q_s \geq V(b,s) \quad \forall b,s$$

$$x(b,s) = 1 \Rightarrow q_b + q_s = V(b,s)$$



$$V(b_i, s_j) = \max\{b_i - s_j, 0\}$$

$q_b = V_b^*(p) \quad q_s = V_s^*(p)$

at p^{**}, buyers get MP_b
p^*, sellers get MP_s

Total gains: $(b_1 - s_1) + (b_2 - s_2) + (b_3 - s_3)$

← here, $V_b^*(p) = MP_b$
$V_s^*(p) = MP_s$