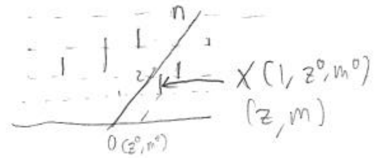
Assignment Model Notes
up to p19 (at most)
Makowski Notes: ch2 + 4

The quasilinear model is sorta an ideal for market socialism.

$x(i, z, m)$



The economy is described by:
$r = (r_1, \ldots, r_n)$ where r_i = mass of type i.
$r_i = \sum_{(z,m)} x(i,z,m)$ ← marginal distribution on i

How do we define a feasible allocation?

Define $x(z,m) \equiv \sum_i x(i,z,m)$ ← marginal distribution on z, m.

$\sum_{(z,m)} \begin{pmatrix} z \\ m \end{pmatrix} x(z,m) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ← Feasible allocation

$x(i,z,m) \geq 0$ → this is a possibility (continuum of agents)

Or we could impose $x(i,z,m) \in \{0,1,2,3,\ldots\}$

Suppose $r = (1,1,\ldots,1)$ and $x(i,z,m) \in \{0,1,2,3,\ldots\}$
$x(i,z,m) = 1$ for exactly one $(z,m) \equiv (z_i, m_i)$
$\Rightarrow \sum_i \begin{pmatrix} z_i \\ m_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Thus, this is just a special case

PTE for $r = (r_1, \ldots, r_n)$: $(x, p) \ni x$ is feasible and
$x(i,z,m) > 0 \Rightarrow v_i(z) + m \overset{\text{budget balancing}}{=} v_i(z) - pz \overset{\text{optimizing}}{=} v_i^*(p)$ (everyone is optimizing)

$r = (r_1, \ldots, r_n)$. Forget about m temporarily

$x(i,z)$

$\sum_z x(i,z) = r_i$

$x(z) \equiv \sum_i x(i,z)$

$\Rightarrow \sum_z z\,x(z) = 0$

$h(r,0) = \max \sum_{i,z} v_i(z)\, x(i,z)$ st z satisfies $\sum_i x(i,z) = r_i$

$\sum_z z\, x(z) = 0$

Consider $h(r,0)$ vs. $h(2r,0)$. $2h(r,0) = h(2r,0)$

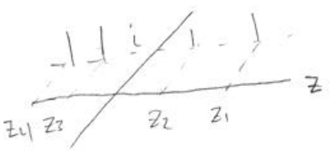
Suppose $x(i,z) \in \{0,1,2,\dots\}$. Then $2h(\vec{1},0) \le h(2\cdot\vec{1},0)$

If v_i concave $\forall i$, then $2h(\vec{1},0) = h(2\cdot\vec{1},0)$

If we eliminate $x(i,z) \in \{0,1,2,\dots\}$, we don't need concave v_i's.

└ convexifying effect of large numbers.

$\hat{v}_i(z) = \max \{ \sum \lambda_k v_i(z_k) : \sum \lambda_k z_k = z \quad \sum \lambda_k = 1, \lambda_k \geq 0 \}$



Consider $r_i = 1$

Then we can "break up" type i and, in some sense concavify v_i. "Convexifying effect of large numbers"

Concavity says $v_i[\sum \lambda_k z_k] \geq \sum \lambda_k v_i(z_k) \Rightarrow$ No benefit to breaking them up if v_i concave.

$\overline{\overline{}}$

$h(\underset{\in \mathbb{R}^n}{r}, \underset{\in \mathbb{R}^\ell}{0})$.

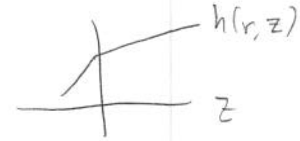
Consider $h(r + 1_i, 0)$ where $1_i = (0,0,\dots,0,\overset{i}{1},0,\dots,0)$

$\underbrace{h(r+1_i,0) - h(r,0)}$ ≥ 0

this is a complex relationship which concerns $v_i \; \forall i, v_j \; \forall j$

$\overline{\overline{}}$

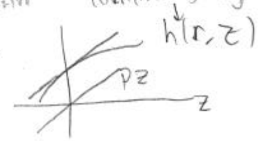
$h(r,\bar{z}) - h(r,0)$ Consider. $\lim_{t \to 0} \frac{h(r,t\bar{z}) - h(r,0)}{t}$



↑ fancy representative agent.

concave by large numbers

Suppose



$\Rightarrow \lim_{t \to 0} \frac{h(r,t\bar{z}) - h(r,0)}{t} = p\bar{z}$

This is the meaning of perturbing the $\bar{z}$ constraint.

Marginal product of commodities

$h(r,\bar{z}) - h(r,0) < p\bar{z}$

$v_I(0) - v_{I\setminus\{i\}}(0) \leftarrow MP_i$ in old framework.

Let $r = (1, \ldots, 1) = \underline{1}$

$h(1, 0) - h(1 - 1_i, 0) \equiv MP_i(\vec{1})$ $\quad MP_i \geq v_i^*(p)$

Consider $h(2 \cdot \vec{1}, 0) - h(2 \cdot \vec{1} - 1_i, 0) = MP_i(2 \cdot \vec{1})$

$MP_i(\vec{1}) \geq MP_i(2 \cdot \vec{1}) \geq \ldots \geq MP_i(k \cdot \vec{1}) \geq v_i^*(p) \; \forall i$

Claim: $MP_i(k \cdot \vec{1}) \xrightarrow{k \to \infty} v_i^*(p) \; \forall i$

This requires the flattening effect of large numbers

$-pz_i \leq v_{-i}(-z_i) - v_{-i}(0) \iff MP_i \geq v_i^*(p)$

perturb the aggregate endowment

$D_z h(r, 0; \bar{z}) = p\bar{z}$

For an individual: $D_z h(r, 0; -z_i) = -pz_i \leftarrow$ exactly reflects social opportunity cost

L With large numbers, there are flats.