- PCE vs PTE
  - Formulas
  - Picture
- Producers
- Another Formulation

PTE of $\underline{v} = (v_i)$ : $[z_i]$, $p$, $[m_i]$

individual budget balance $p z_i + m_i = 0$

mkt. clearance $\sum z_i = 0 \Rightarrow \sum m_i = 0$

utility max $v_i(z_i) - p z_i = v_i^*(p) \quad \forall i$

Those can be summarized by the existence of $(z_i)$ and $p$ satisfying:

$$v_I(0) = \sum_{i=1}^{n} v_i(z_i) \quad \text{and} \quad p \in \partial v_I(0)$$

PCE is a strengthening of PTE

PCE = PTE: $\{[z_i], p\}$ +

(I) $-p z_i = v_{-i}(-z_i) - v_{-i}(0) \ \forall i$ (social opportunity cost/benefit) SOC

or

(II) $v_i^*(p) = MP_i \equiv v_I(0) - v_{-i}(0) \quad \forall i$ (full appropriation) FA

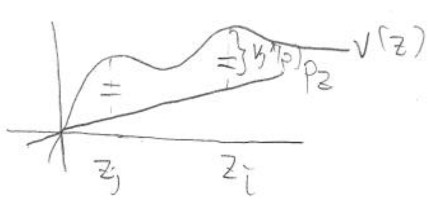
or

(III) $p \in \partial v_{-i}(0)$ [$\exists$ PTE for $\underline{v}_{-i}$ $\{[z_i'], p'\}$ such that $p = p'$] $\forall i$.

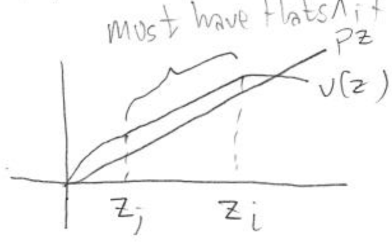
L prices do not change. (perfectly elastic demand and supply) PEDS

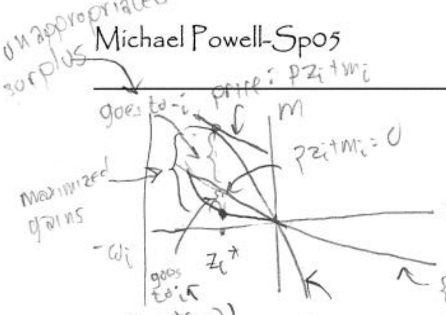
Obviously, $\sum_{j \neq i} z_j = 0$

L No one's welfare changes when you remove $i$, since prices do not change.




must have flats parallel to pz if concavified






$\hookrightarrow$ This is not PCE.

In order to make this PCE, we must flatten the social indifference curve along the region between the origin and the part where trade would occur (and slope = price ratio)

Producers:

$v_j(z_j) = -c_j(z_j)$

$0 \le c_j(z_j) \equiv$ cost of producing $z_j$ = minimum number of units of $m$ required to "produce" $z_j$.

Suppose we have $Y_j \subset \mathbb{R}^{\ell+1}$

$$c_j(z) = \begin{cases} \inf\{m : (z_j, m_j) \in Y_j\} & \text{if } (z_j, m_j') \in Y_j \text{ for some } m' \\ +\infty & \text{otherwise.} \end{cases}$$

Special case: $Y_j = Z_j \times \{0\}$ $\rightarrow$ money commodity is neither input or output.

$Z_j \subset \mathbb{R}^\ell$

$\Rightarrow c_j(z_j) < +\infty \Rightarrow c_j(z_j) = 0$

This corresponds to a production set in Theory of Value.

$$\pi_j(p) = v_j^*(p) = \sup_{z_j}\{v_j(z_j) - p\cdot z_j\} = \sup_{z_j}\{-p\cdot z - c(z_j)\} \quad \left(= \sup_z\{-p\cdot z\} \text{ in the } c(z_j) = \left\{\begin{smallmatrix}0\\+\infty\end{smallmatrix}\right. \text{ case}\right)$$

---

$v_i \sim (v_{i\,\text{consumer}}, v_{j\,\text{producer}})$ where

$v_{i\,con}(z_i) > -\infty \Rightarrow z_i \ge -\omega_i \in \mathbb{R}^\ell_+$

$v_{j\,pro}(z_j) = -c(z_j)$

$V_i(z) = \sup_y \{ v_{icon}(y) + v_{ipro}(z-y) \}$ ← this is like Robinson Crusoe economy.

$v_i^*(p) \geq v_{icom}^*(p) + v_{ipro}^*(p$

In a competitive world, profits are rents on scarce technology.

In PTE:

$v_{ipro}^*(p) \leq v_I(0) - v_{I \setminus \{ipro\}}(0)$

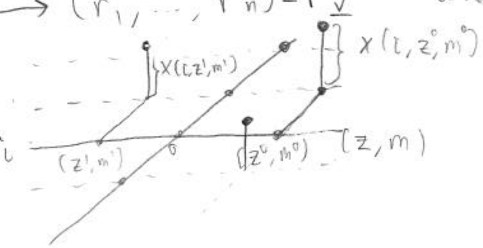
In PCE:

$v_{ipro}^*(p) = v_I(0) - v_{I \setminus \{ipro\}}(0)$

another formulation: (Reformulate the description of allocations)

$\underline{V} = (v_1, \dots, v_n) \longrightarrow (r_1, \dots, r_n) = r_{\underline{V}}$ where $r_i = \# v_i \in \mathbb{N} \cup \{0\}$

$X(i, z, m)$
↑
number of people of type $i$ trading $(z, m)$
└ mass function




Feasible for $r_{\underline{V}} = (r_1, \dots, r_n)$

(1) $\sum_{z,m} x(i, z, m) = r_i$

Define $x(z, m) \equiv \sum_i x(i, z, m)$

(2) $\sum_{z,m} \binom{z}{m} \underbrace{x(z,m)}_{\substack{\text{scalar} \\ \in \mathbb{R}}} = \binom{0}{0}$ where $\binom{z}{m} \in \mathbb{R}^{l+1}$

"measures with finite support"
└ allows for simple summations.

---

$(x, p)$ is a PTE for $r_{\underline{V}}$ if

I. $x$ is feasible. (satisfies (1) and (2))

II. $x(i, z, m) > 0 \Rightarrow m = -pz$ (everyone is on their budget constraint)

III. $x(i, z, m) > 0 \Rightarrow v_i(z) + m = v_i^*(p)$

---

Let $r_{\underline{V}} = (1, 1, \dots, 1)$ and $x(i, z, m) \in \{0, 1, 2, 3, \dots\} \Rightarrow \forall i \; \exists! \, (z, m) \ni x(i, z, m) > 0$

Now, $\sum_{z,m} \binom{z}{m} x(z, m) = \binom{0}{0}$ is the same as $\sum z_i = 0$

$\sum m_i = 0$

We have recovered the finite model.

Now we can, by reallocating weights, talk about the continuum economy.

When $r=(1,\dots,1)$, we used the notion of $v_I(0)$ to denote maximum gains from trade

When $r=(r_1,\dots,r_n)$, we will use $g(r)$ to denote the maximum gains from trade.

We will consider:

$$\nabla g(r) = \left( \frac{\partial g(r)}{\partial r_i} \right) = \left( MP_i \right)$$

$\Rightarrow$ the calculus will apply in the continuum economy.

In discrete economy, this corresponded to

$$v_I(0) - v_{I \setminus \{i\}}(0) = g(1,\dots,1) - g(1,\dots,1,\overset{i}{0},1,\dots,1)$$

When $\nabla g(r)$ exists, $MP_i = v_i^*(p)\ \forall i$