Value and distribution - Theory of Value
↳ prices of consumer goods (outputs) } commodities
↳ prices of producer goods (inputs)

"Distribution" also refers to payments (income/wealth) to individuals

Individual wealth $w_i(p) = p \cdot \omega_i + \sum_j \theta_{ij} \pi_j(p)$ - determines "distribution" of wealth.

L choose definition of ownership (in particular, θ_{ij})

$w_i(p) = \alpha_i w(p)$

$= \alpha_i \left(p \cdot \omega + \sum_j \pi_j(p)\right)$ ← This also works just fine.

<u>Want to</u> tighten the connection between ownership and reward.

Commodity infinitesimal analysis of rewards at PTE

$$Dv_i(z_i; y) = \lim_{t \to 0} \frac{v(z_i + ty) - v_i(z_i)}{t}$$ - homogeneous of degree 1 with respect to y.

L directional derivative.

assume v_i is differentiable. Then $\exists\, p \ni py = Dv_i(z_i, y)\ \forall y$

L locally, everything looks linear.

$\underbrace{Dv_I(0; -y)}_{\text{maximum gains from trade}} = \underbrace{-py}_{\text{cost to society}}$ ← infinitessimally, this holds.

Suppose $py > 0 \Rightarrow$ would have to pay $m_i = -py$

Suppose $py < 0 \Rightarrow$ would have to be compensated $m_i = -py$

$(v(z_i), p)$ $m_i = -pz_i$ ← PTE

$\Rightarrow p \in \partial v_i(z_i)$

If $p \in \partial v_i(z_i)\ \forall i$, then $p \in \partial v_I(0)$

where $v_I(0) = \sum v_i(z_i)$ where $\sum z_i = 0$ and this maximizes.

We want to take this idea and apply it non-infinitessimally. In order to do this, we need to tighten the definition of ownership.

Individual (non-infinitesimal) analog of this:

(1) What is the social value of z_i in PTE?

(2) Is it $-pz_i$? No, but there is an inequality.

(1): Recall: $I = \{1, \ldots, n\}$

$$v_I(\bar{z}) = \sup\left\{ \sum_{i=1}^{n} v_i(z_i) \;\middle|\; \sum z_i = z \right\}$$

$$v_{-i}(\bar{z}) = \sup\left\{ \sum_{j \neq i} v_j(z_j) \;\middle|\; \sum_{j \neq i} z_j = z \right\}$$

$$v_{-i}(0) = \sup\left\{ \sum_{j \neq i} v_j(z_j) \;\middle|\; \sum_{j \neq i} z_j = 0 \right\}$$

Define $\Delta v_{\cdot}(z) \equiv v_{-i}(z) - v_{-i}(0)$

$v_I(0) = \sup_z \{ v_i(z) + v_{-i}(-z) \}$ where $-i$ is "everyone else"

$\downarrow$

If $(z_i), p$ is a PTE, then $v_I(0) = v_i(z_i) + \sum_{j \neq i} v_j(z_j)$

$= v_i(z_i) + v_{-i}(-z_i)$ since $\sum_{j \neq i} z_j = -z_i$

$p \in \partial v_j(z_j)$, $j \neq i$ $\Rightarrow$ $p \in \partial v_{-i}(-z_i)$

$p \in \partial v_i(z_i)$

$\hookrightarrow v_{-i}(-z_i) - (p(-z_i)) \geq v_{-i}(0) - p \cdot 0$

[Recall: $v^*(p) = \sup_{z'} \{ v(z') - p \cdot z' \}$

$v(z) - p \cdot z = v^*(p) \Leftrightarrow p \in \partial v(z)$]

$$** \Rightarrow \quad -pz_i \leq v_{-i}(-z_i) - v_{-i}(0) \longleftarrow \text{gain to } -i.$$

(This is important. (infinitesimally, w/differentiability, this is "=")

compensation to i

$MP_i \equiv v_I(0) - v_{-i}(0)$ ← amt. by which the total gains from trade change by getting rid of i.

$= \sup\{v_i(z) + v_{-i}(-z)\} - v_{-i}(0)$

$= \sup_z\{v_i(z_i) + \Delta v_{-i}(-z_i)\} \underset{\text{in PTE}}{=} v_i^*(p) + \Delta v_{-i}(-z_i) - pz_i$

$v_i^*(p) = \sup_z\{v_i(z) - pz\} \overset{\text{PTE}}{=} v_i(z_i) - pz_i$

MP_i vs $v_i^*(p)$

$[-pz_i \leq \Delta v_{-i}(-z_i) = MP_i - v_i^*(p) - pz_i \Rightarrow MP_i \geq v_i^*(p)]$

The decisive factor for this was $-pz_i \leq v_{-i}(-z_i) - v_{-i}(0)$

└ Your reward is based only on others' valuation of your trade.

Recall: $\sum v_i(z_i) \leq \sum v_i^*(p) \quad \forall p, \forall z \ni \sum z_i = 0$

In PTE, $\sum v_i(z_i) = \sum v_i^*(p) = v_I(0)$

** └interesting $\Rightarrow \sum MP_i \geq v_I(0) \Rightarrow$ cannot have full appropriation
↳ will run out of the pie!

This course will focus on "when is $\sum MP_i = v_I(0)$?"

└ Perfect competition has $MP_i = v_i^*(p)\ \forall i \Rightarrow$ full appropriation.

$MP_i > v_i^*(p) \Rightarrow$ There are opportunities for clever people.

Result: Unless you get full appropriation, the non-cooperative approach will lead to a smaller pie.

For a monopolist, $MP_i > v_i^*(p)$

If $-pz_i = v_{-i}(z_{-i}) - v_{-i}(0)$, then society is indifferent b/t having i in society and not.

The PTE $(z_i), p$ is perfectly competitive

$\iff MP_i = v_i^*(p) \quad \forall i$ total gains = rewards $\forall i$

$\iff -pz_i = v_I(-z_i) - v_{-i}(0) \quad \forall i$ money gain = social opportunity

$\iff p \in \partial v_{-i}(0) \quad \forall i$ (perfectly elastic demand and supply)

∟ no one has the ability to change prices.

$\underline{V} = (v_1, \ldots, v_n)$

$p \in \partial v_I(0) \leftarrow$ This is a very complicated representative agent.

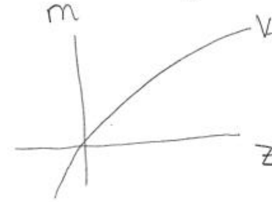
$\underline{V}_{-i} = (v_j)_{j \neq i}$ $\quad p \in \partial v_{-i}(0) \qquad \sum_{j \neq i} v_j^*(p) = v_{-i}(0)$

∟ indirect utility is <u>only</u> a function of p, so if p does not change, no one is better/worse off.

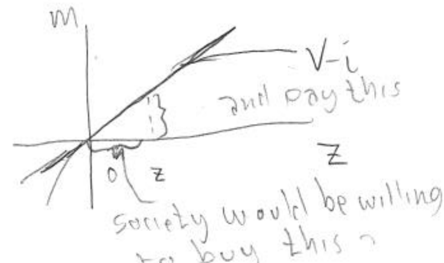
If there are only two people, if the size of the pie is >0, then $MP_1 = v_I(0)$ and $MP_2 = v_I(0)$ <u>and</u> $MP_1 + MP_2 = v_I(0) > 0$

⟶✕ There is no competition here. Only complementarity.

$v_i^*(p) = \sup\{v(z) - pz\}$



Each person must sell on society's "flat spot!"



If we have a continuum of consumers, then each trade is infinitesimally small. (can't affect prices since too small)

∟ This is not possible if everyone has strictly concave v-fcn,

∟ Need some substitutability.