Use QL model to highlight the commodity versus the individual as a margin of analysis.

$$\sup\Big\{ \sum_{i=1}^{n} v_i(z_i) \,\Big|\, \underbrace{\sum_{i=1}^{n} z_i = 0}_{\text{aggregate resource constraint}} \Big\} \equiv v_I(0) \longleftarrow \text{Pareto optimal}$$

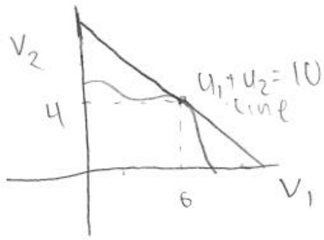
$\underline{v} = (v_1, \dots, v_n)$ $\qquad I = \{1, \dots, n\}$

eg $v_I(\bar{z}) = \sup\{ \sum_{i=1}^{n} v_i(z_i) \mid \sum_{i=1}^{n} z_i = \bar{z} \}$ ← gift from outside, etc.

$V_I : F^n \times \mathbb{R} \to \mathbb{R}$ where F is the functional space.

$$\sup\Big\{ \sum_i [v_i(z_i) + m_i] \,\Big|\, \sum_i z_i = 0,\ \sum_i m_i = 0 \Big\} = v_I(0)$$

$v_I(0)$ is the max size of the social pie



$p \in \partial v(z)$ - inverse dmd (subdifferential)

$v_i^*(p) = \sup_{z_i} \{ v(z_i) - p \cdot z_i \}$

no gifts (superfluous by defn of $v^*(p)$)

PTE: $[(z_i), p, \overbrace{pz_i + m_i = 0}]$ ($m_i = -pz_i \ \forall i$ since $pz_i + m_i = 0$)

such that $\sum z_i = 0 \ \forall i$ [$\sum m_i = \sum -pz_i = -p \sum z_i = -p\, 0 = 0 \ \forall i$]

and $v_i(z_i) - pz_i = v_i^*(p) \ \forall i$

Claim: This PTE is Pareto optimal

Prop Fix $(z_i) \ni \sum_i z_i = 0$ and $\underline{\underline{\text{any}}}$ p, $\underbrace{\sum_i v_i(z_i)}_{\text{soln primal problem}} \le \underbrace{\sum_i v_i^*(p)}_{\text{soln dual prob}}$

Pf: $v_i^*(p) \ge v_i(z_i) - pz_i$

$$\sum_i v_i^*(p) \ge \sum_i [v_i(z_i) - pz_i] = \sum_i v_i(z_i) \quad \text{since } \sum -pz_i = -p \sum z_i = 0$$

Suppose we found some z_i and $p \ni$

$\sum_i v_i(z_i) = \sum_i v_i^*(p)$. Then z_i, p are such that $\sum_i v_i(z_i) = v_I(0)$

Prop: If $\sum v_i^*(p) = \sum v_i(z_i)$, then $\sum v_i(z_i) = v_I(0)$. That is, (z_i) is PO.

Prop: PTE $\Rightarrow$ PO

$$PO \overset{?}{\Rightarrow} PTE$$

PO : $[(z_i, m_i)] \ni \sum_i v_i(z_i) = 0$
$\sum_i m_i = 0$

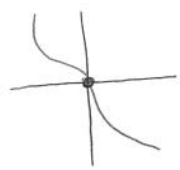
There are two ways this is not true.

1) PTE need not exist

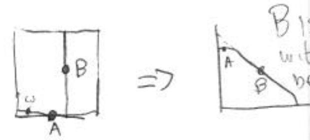
$\underline{V} \longrightarrow v_I(0) \Rightarrow PO$

Can $\hat{v}_I(0) > v_I(0)$? where $\hat{v}_I(0) = \sup\{\sum \hat{v}_i(z_i) \mid \sum z_i = 0\}$

Yes. Consider a one-person economy. It is PO for there to be no trade.



2) PTE exists but $m_i \neq -p z_i$



B is PO, but with ω, cannot be supported as a PTE.

(since in PO, we can split up the pie any way we want. In PTE, we aren't necessarily doing this)

In PTE, people are rewarded for trading by $m_i = -p z_i$.

In PO, $\tau_i + m_i = -p z_i$ $\quad \sum \tau_i = 0$

Production:

$$v_j(z_j) = \begin{cases} 0 & \text{if } z_j \in Z_j \\ -\infty & \text{if } z_j \notin Z_j \end{cases}$$ ← indicator function for the set Z_j

$Z_j \to Y_j$

$$v_j^*(p) = \sup_{z_j} \{ v_j(z_j) - pz_j \}$$

$$= \sup_{z_j} \{ -pz : z \in Z_j \}$$

maximized profit

$-p \cdot z$ = Revenue − cost

$\underline{v} = ((v_i), (v_j)) \qquad \theta_{ij}$

PTE: $[(z_i), (z_j), (m_i),$

Such that $\sum_i z_i + \sum_j z_j = 0$

$$v_j^*(p) = v_j(z_j) - pz_j$$

$$v_i(z_i) + m_i = \sup \{ v_i(z_i') + m_i' \mid pz_i' + m_i' = \sum_j \theta_{ij} v_j^*(p) \}$$

$$= v_i^*(p) + \underbrace{\sum_j \theta_{ij} v_j^*(p)}_{\tau_i}$$

$m_i = -pz_i$ ← reward the individual is getting $\qquad m_i = p(-\overset{\text{i's contribution to society}}{\downarrow}z_i)$

$p \in \partial v_I(0) \Rightarrow v_I(0) = \sum v_i(z_i), \quad \sum_i z_i = 0$

$\Rightarrow p \in \partial v_i(z_i)$

$$\lim_{t \downarrow 0} \frac{v_I(tz_i) - v_I(0)}{t} = pz_i \Rightarrow$$ individual is being rewarded with the social value of his trade.

This is not how things work.

Consider $m_i = -p\overset{\text{contribution}}{\downarrow}z_i$