Typos in 1st week notes
$\Omega \equiv \mathbb{R}^{\ell}_{+}$
$\mathbb{R}^{\ell(m+n)}$ $m \equiv \#$ consumers
$n \equiv \#$ producers

Last page of part I: alienable $\rightarrow$ inalienable

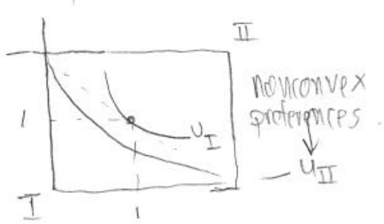
$w(p) = p \cdot \omega + \sup pY$ aggregate wealth for economy E
$w_i(p) = p \cdot \omega_i + \sum_j \theta_{ij} \sup pY_j$ aggregate wealth for economy $\mathcal{E}$
We could have used $w_i^{\alpha}(p) = \alpha_i w(p)$ - Market socialism
$\xi_i(p, w_i(p)) \equiv$ demand correspondence
$\eta_j(p) \equiv$ supply correspondence

With the new wealth functions, we can define:
$\xi_i(p, w_i^{\alpha}(p))$
The equilibria are the same in both situations.

"Replica invariance" - constant returns to scale with respect to people.
$\mathcal{E} \to \omega \quad Y = \sum Y_j$
$2\mathcal{E} \to 2\omega \quad Y + Y$
$k\mathcal{E} \to k\omega \quad \underbrace{Y + Y + \dots + Y}_{k}$

Can we increase per-capita gains by replicating this economy? If no, then the economy is replica invariance

Replica invariance gives price-taking equilibrium
Replica invariance $\not\Rightarrow$ CRS



Suppose $\omega_1 = \omega_2 = (1,1)$
$\Rightarrow \omega = \omega_1 + \omega_2 = (2,2)$
There is no price-taking equilibrium for this model.

With $2\mathcal{E}$, $\omega = (4,4)$
The two type I's consume $(1,1)$
The two type II's consume $(0,2), (2,0)$ respectively } an equilibrium exists here.

If we define $2\mathcal{E} \equiv \mathcal{E}'$, then $\mathcal{E}'$ is replica invariant.

If p is an equilibrium price for $\mathcal{E}$, it is an eq. price for $2\mathcal{E}$ if $\mathcal{E}$ is replica invariant.

$$\sum_i \xi_i(p, \omega(p)) - \sum_j \eta_j(p) = \omega$$

$$\Leftrightarrow \quad 2\sum_i \xi_i(p, \omega(p)) - 2\sum_j \eta_j(p) = 2\omega$$

Increasing $\overset{\text{Unbounded}}{\text{returns}}$ to scale $\Rightarrow \neg$(Replica invariance)
with respect to commodities

Quasi-linear GE: (subset of posted notes)

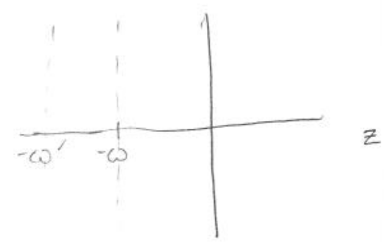
Consumer: (we aren't writing the index subscripts)

$$U(\omega + z, m) = u(\omega + z) + m$$
money
$$= v(z) + m$$

$\omega + z \in \mathbb{R}^\ell \qquad \omega + z \geq 0$

$m_i \in \mathbb{R}$

where $v(z) \equiv \begin{cases} u(\omega + z) & z \geq \omega \\ -\infty & \text{else} \end{cases}$



[describes tastes and initial endowment
["can't supply more than I have"

z (+ = comes in)
(− = goes out)

We will normalize such that $p_m = 1$

The prices that remain are the $p \in \mathbb{R}^\ell_+$

Budget constraint: $p \cdot z$ – net expenditures associated with trade z.

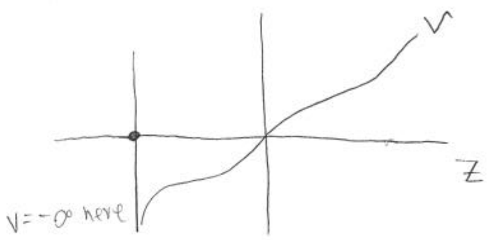
Endowment of money commodity will be zero. m can be positive or negative.

Budget constraint:

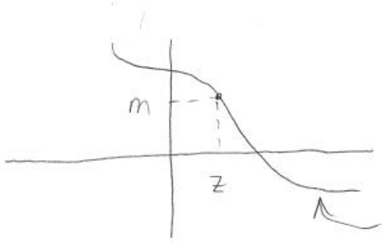
$pz + \cancel{p_m} m = 0 \Rightarrow p \cdot z + m = 0$

Define $A(z,m) = \{(z', m') \mid v(z') + m' \geq v(z) + m\}$ - at least as good as set

Let $l = 1$



- we drew this with $v(0) = 0$. This is wlog. In particular, 0 is a feasible trade.



Once we know what one indifference curve looks like, we know all of them

$A(z,m)$ is always convex iff v is concave.

Demand

$d(p, w) = (z, m)$

$pz + m = w$

$d(p, 0) = (z', m')$

$\Rightarrow$ No income effects